[YOUR SCHOOL HERE]

NATIONAL 5 MATHEMATICS REVISION BOOKLET

Contents

What's in this booklet

- This booklet will help you prepare for your N5 Mathematics exam.

- There are 5 sections altogether, covering **Operational** and **Reasoning** skills as well as providing opportunities to practise non-calculator work.

- The questions in Sections A and B cover **'Operational Skills'**. **Operational Skills** require you to **demonstrate a knowledge and understanding of key facts and procedures.**

- **Operational Skills** have been **organised around 4 themes**: Algebra, Geometry, Trigonometry and Statistics & Number.

- A **description** of each **Operational Skill** is given on **Page 5** e.g. **Algebra Skill 9** (**A9**) is 'changing the subject of a formula'.

- The questions in Sections C and D cover **'Reasoning Skills'**. As well as using **Operational Skills**, a reasoning question will require you to select a strategy or interpret/communicate clearly.

- On **Page 4** the questions from Sections A to D have been mapped against the **Operational Skills.**

- You can **assess your progress** using the table on **Page 6**

- **Section E** contains questions on each **Operational Skill**. This section will be useful if you need more practice with a particular skill(s)

Summary of questions by skill

Algebra					Geometry					Trigonometry					Statistics & Number				
	ONC	OC	RNC	RC		ONC	OC	RNC	RC		ONC	OC	RNC	RC		ONC	OC	RNC	RC
A1	4,6, 22,37	12,18	6,12 17,21	29,39 49	G1	42				T1	40			23,30 46	N1	14,19 23,32 49,51		7,9 31	
A2	11,22 27,43				G2		8,20 31		4,18 38,44	T2		32		8,15 23,30 46	N2	14,22 31,39	1,22	11,35	5
A3	41,44 50				G3		3,7		21,26 31,50	T3				2,9 28 35,43	N3		1,6 13 17,22 28		
A4	36,43				G4	23,32		7,31, 34	7,14 20,26 41	T4		4,5 15	16,26 27,42	6,12 14 36 48	N4	18,24	6,9 11 14 17 25,26 27,30		1,3 16
A5	30		10,28 30,32		G5			34	7,20	T5				36,48	N5	2,15 16,21 35,42 50	16	8,13	
A6	12,29 38		3,15 23	13	G6		10,21 24		10,17 19						S1			1	11,25 32,47
A7	4,13, 42	29	4,15 20,22 27,30	4,10 21	G7										S2				33
A8	9,47		2,14 19,24 33	40	G8	48													
A9	3,7, 17,25 28			22,50	G9	48,49													
A10	45																		
A11	46																		
A12	8,45 46		25,29	51															
A13	46	2,13 19,23 28	5,9 18,25 29	2,17 24,29 37,45 49,51															

National 5 Mathematics – Skills Summary

Algebra		Geometry		Trigonometry		Statistics & Number	
A1	Multiply out brackets and simplify	G1	Find the gradient of a straight line	T1	Graphs of trigonometric functions	N1	Work with surds: simplify expressions; rationalise denominator
A2	Factorise: common factor, difference of 2 squares, trinomials and combinations	G2	Calculate the length of arc or area of sector	T2	Solve trigonometric equations; establish simple identities	N2	Use the laws of indices, including calculations using scientific notation
A3	Complete the square	G3	Calculate the volume of standard solids and composite shapes	T3	Calculate the area of a triangle using trigonometry (formula given)	N3	Round to a given number of significant figures
A4	Reduce an algebraic fraction to its simplest form	G4	Apply Pythagoras' Theorem in complex situations, including converse and 3D	T4	Use sine/cosine rules to determine a side/angle	N4	Work with percentages: appreciation/depreciation and 'reversing'
A5	Add, subtract, multiply and divide algebraic fractions	G5	Find an angle using the properties of circles, triangles, polygons etc	T5	Use bearings with trigonometry	N5	Operations (and combinations) on fractions/mixed numbers
A6	Find/interpret the equation of a straight line e.g. $y - b = m(x - a)$, $y = mx + c$ etc	G6	Use similarity with lengths, areas and volumes			S1	Calculate/interpret interquartile range and standard deviation
A7	Solve linear equations and inequations	G7	Add/subtract vectors in 2D using directed line segments			S2	Find and use the equation of a best-fitting straight line
A8	Work with simultaneous equations: solve graphically or algebraically	G8	Determine coordinates of a point from a diagram in 3D				
A9	Change the subject of a formula	G9	Add/subtract vectors in 2D/3D using components; find magnitude of a vector				
A10	Recognise/determine the equation of a quadratic function from its graph						
A11	Sketch the graph of a quadratic function						
A12	Identify nature and coords of TP,equation of axis of symmetry of a quadratic						
A13	Work with quadratic equations: factorise; graph; formula; discriminant						

National 5 Mathematics: Self-Assessment

Algebra		Geometry		Trigonometry		Statistics & Number	
A1		G1		T1		N1	
A2		G2		T2		N2	
A3		G3		T3		N3	
A4		G4		T4		N4	
A5		G5		T5		N5	
A6		G6				S1	
A7		G7				S2	
A8		G8					
A9		G9					
A10							
A11							
A12							
A13							

Section A

Operational Skills – No Calculator (ONC)

1. Evaluate

$$86{\cdot}5 - 3{\cdot}651 \times 20.$$

2. Evaluate

$$\frac{1}{2} \div 2\frac{2}{3}.$$

3. Change the subject of the formula to r.

$$A = 4\pi r^2.$$

4. (*a*) Expand and simplify

$$(2x - 5)(x^2 + 3x - 7).$$

(*b*) Solve the inequality

$$4x - 5 \leq 7x - 20.$$

5. Evaluate

$$7{\cdot}2 - 0{\cdot}161 \times 30.$$

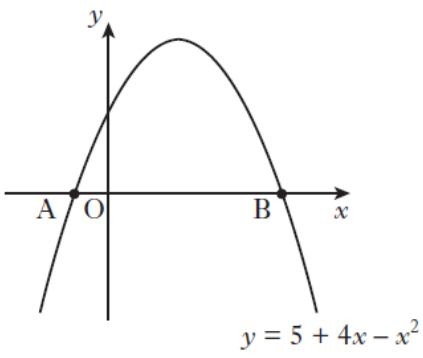
6. Expand and simplify

$$(3x - 2)(2x^2 + x + 5).$$

7. Change the subject of the formula to m.

$$L = \frac{\sqrt{m}}{k}$$

8. The diagram shows part of the graph of $y = 5 + 4x - x^2$.



A is the point (–1, 0).

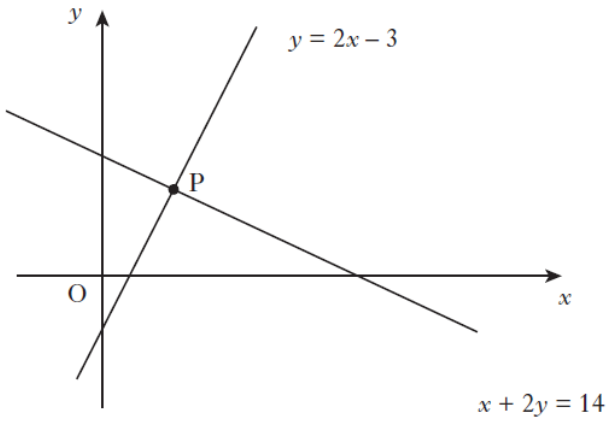
B is the point (5, 0).

(*a*) State the equation of the axis of symmetry of the graph.

(*b*) Hence, find the maximum value of $y = 5 + 4x - x^2$.

9. The graph below shows two straight lines.

- $y = 2x - 3$
- $x + 2y = 14$



The lines intersect at the point P.

Find, **algebraically**, the coordinates of P.

10. Evaluate

$$2{\cdot}4 + 5{\cdot}46 \div 60.$$

11. Factorise fully

$$2m^2 - 18.$$

12. Given that $f(x) = 5 - x^2$, evaluate $f(-3)$.

13. Solve the equation

$$3x + 1 = \frac{x - 5}{2}.$$

14. (*a*) Simplify $2a \times a^{-4}$.

(*b*) Solve for x, $\sqrt{x} + \sqrt{18} = 4\sqrt{2}$.

15. Evaluate

$$40\% \text{ of } £11{\cdot}50 - £1{\cdot}81.$$

16. Evaluate

$$\frac{2}{5} \div 1\frac{1}{10}.$$

17. Change the subject of the formula to s.

$$t = \frac{7s + 4}{2}.$$

18. Cleano washing powder is on special offer.

Each box on special offer contains 20% more powder than the standard box.

A box on special offer contains 900 grams of powder.

How many grams of powder does the standard box contain?

19. (*a*) Simplify $\sqrt{2}\times\sqrt{18}$.

(*b*) Simplify $\sqrt{2}+\sqrt{18}$.

(*c*) Hence show that $\dfrac{\sqrt{2}\times\sqrt{18}}{\sqrt{2}+\sqrt{18}} = \dfrac{3\sqrt{2}}{4}$.

20. Evaluate

$$(846 \div 30) - 1{\cdot}09.$$

21. Evaluate

$$4\frac{1}{3} - 1\frac{1}{2}.$$

22. (*a*) Factorise

$$x^2 - 4y^2.$$

(*b*) Expand and simplify

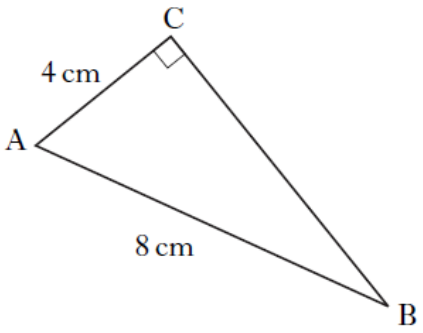
$$(2x - 1)(x + 4).$$

(*c*) Expand

$$x^{\frac{1}{2}}\left(3x + x^{-2}\right).$$

23. In triangle ABC:

- angle ACB = 90°
- AB = 8 centimetres
- AC = 4 centimetres.



Calculate the length of BC.

Give your answer **as a surd in its simplest form**.

24. This year, Ben paid £260 for his car insurance.
This is an increase of 30% on last year's payment.

How much did Ben pay last year?

25. A formula used to calculate the flow of water in a pipe is

$$f = \frac{kd^2}{20}.$$

Change the subject of the formula to d.

26. Evaluate

$$24{\cdot}7 - 0{\cdot}63 \times 30.$$

27. Factorise fully

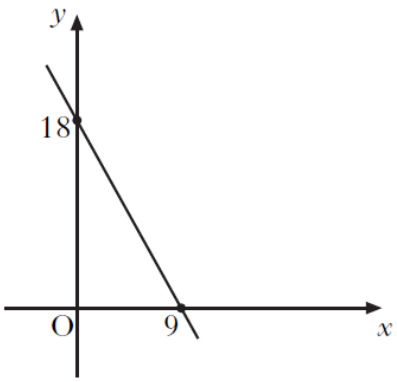
$$5x^2 - 45.$$

28.

$$W = BH^2.$$

Change the subject of the formula to H.

29. A straight line cuts the x-axis at the point (9, 0) and the y-axis at the point (0, 18) as shown.



Find the equation of this line.

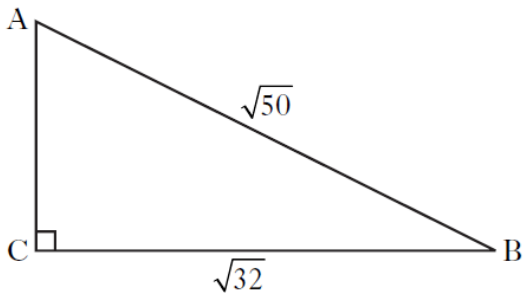
30. Express as a single fraction in its simplest form

$$\frac{1}{p}+\frac{2}{(p+5)}.$$

31. Simplify

$$m^3 \times \sqrt{m}.$$

32. A right angled triangle has dimensions as shown.



Calculate the length of AC, leaving your answer as a surd **in its simplest form**.

33. Given that

$$x^2 - 10x + 18 = (x - a)^2 + b,$$

find the values of a and b.

34. Evaluate

$$6{\cdot}04 + 3{\cdot}72 \times 20.$$

35. Evaluate

$$3\tfrac{1}{6} \div 1\tfrac{2}{3}.$$

36.

$$P = \frac{2(m-4)}{3}$$

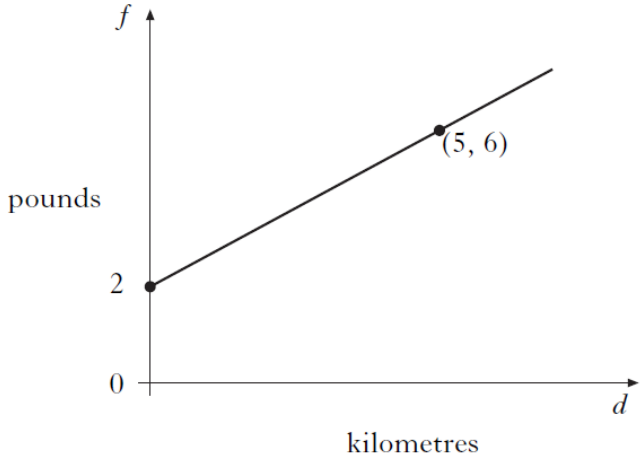
Change the subject of the formula to m.

37. Remove brackets and simplify

$$(2x+3)^2 - 3(x^2-6).$$

38. A taxi fare consists of a £2 "call-out" charge **plus** a fixed amount per kilometre.

The graph shows the fare, f pounds for a journey of d kilometres.



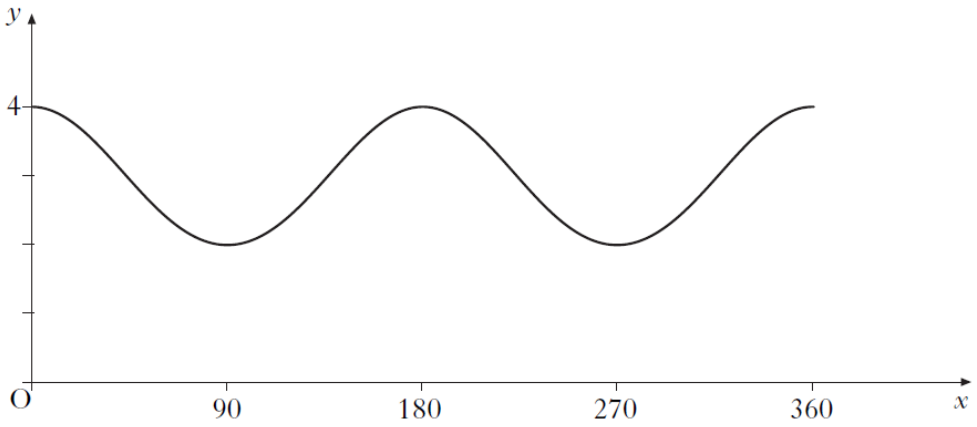
The taxi fare for a 5 kilometre journey is £6.

Find the equation of the straight line in terms of d and f.

39. Remove brackets and simplify

$$a^{\frac{1}{2}}(a^{\frac{1}{2}} - 2).$$

40. Part of the graph of $y = \cos bx^\circ + c$ is shown below.



Write down the values of b and c.

41. Express $x^2 + 4x + 8$ in the form $(x + a)^2 + b$.

42. A straight line passes through $P\left(\frac{3}{10}, 1\frac{7}{12}\right)$ and $Q\left(1\frac{1}{10}, 2\frac{1}{3}\right)$. Find the gradient of PQ.
Give your answer as a fraction in its simplest form.

43. (a) Factorise $x^2 - 2x - 15$.

(b) Hence, simplify

$$\frac{2x-10}{x^2-2x-15}, \quad x \neq -3, 5$$

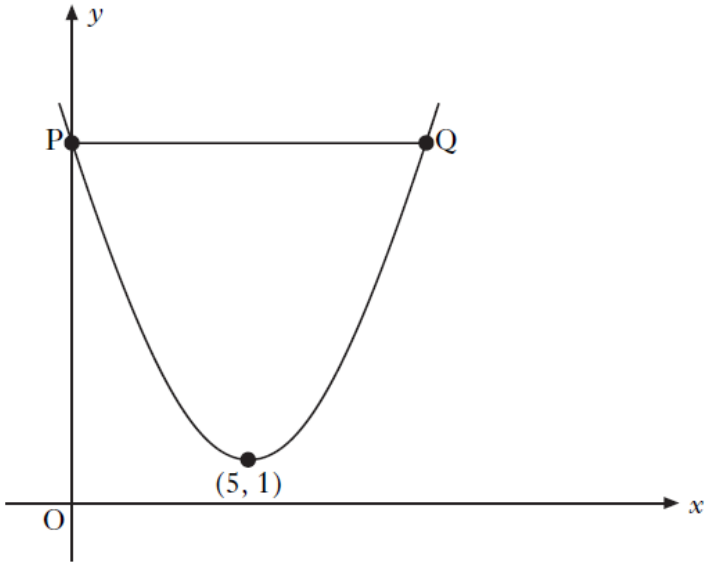
44. (a) Express $x^2 - 2x + 3$ in the form $(x - a)^2 + b$.

A quadratic function is given by $f(x) = x^2 - 2x + 3$.

(b) Write down the minimum value of $f(x)$.

45. The graph below shows part of a parabola with equation of the form

$$y = (x + a)^2 + b.$$



(*a*) State the values of a and b.

(*b*) State the equation of the axis of symmetry of the parabola.

(*c*) The line PQ is parallel to the x-axis.
Find the coordinates of points P and Q.

46. A quadratic function has equation $f(x) = (x + 3)^2 - 16$.

(a) Write down the coordinates of the turning point on the graph of $y = f(x)$.

(b) (i) Find where the graph of $y = f(x)$ meets the y-axis.

(ii) Show that $f(x) = x^2 + 6x - 7$.

(c) Find the coordinates of the points where the graph of $y = f(x)$ meets the x-axis.

(d) Sketch the graph of $y = f(x)$, showing clearly the turning point and where the graph meets the x and y-axes.

47. Solve the system of equations

$$\begin{aligned} 2a + 9b &= 4 \\ a - 3b &= -3 \end{aligned}$$

48. P has coordinates $(3, -4, 1)$.

$\overrightarrow{PQ} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$. Find the coordinates of Q.

49. Given $\overrightarrow{AB} = \begin{pmatrix} -8 \\ 4 \end{pmatrix}$ calculate $\left|\overrightarrow{AB}\right|$. Give your answer as a surd in its simplest form.

50. Express $x^2 + 3x + 2$ in the form $(x + a)^2 + b$.

51. Express $\frac{4}{\sqrt{7}}$ as a fraction with a rational denominator.

[END OF SECTION A]

Section A - Answers

1. $\bullet^1$ must involve a multiplication followed by a subtraction

$\bullet^2$ $13 \cdot 48$

2. $\bullet^1$ $\frac{1}{2} \times \frac{3}{8}$

$\bullet^2$ $\frac{3}{16}$

3. $\bullet^1$ $r^2 = \frac{A}{4\pi}$

$\bullet^2$ $r = \sqrt{\frac{A}{4\pi}}$

4. $\bullet^1$ any 3 correct terms

$\bullet^2$ a further 3 correct terms

$\bullet^3$ $2x^3 + x^2 - 29x + 35$

$\bullet^1$ $-3x$ or $3x$

$\bullet^2$ -15 or 15

$\bullet^3$ $x \geq 5$ or $5 \leq x$

5. • $4 \cdot 83$

• $2 \cdot 37$

6. • $6x^3 - x^2 + 13x - 10$

7. • $\sqrt{m} = kL$

• $\mathrm{m} = (\mathrm{kL})^2$ or $\mathrm{k}^2\mathrm{L}^2$

8. • 2

• $x = 2$

• $y = 5 + 4(2) - 2^2$

• 9

9. • $2y = -x + 14$

• $4y = -2x + 28$

• $y = 5$

• (4,5)

11. • $2(m^2 - 9)$

• $2(m-3)(m+3)$

12. • $5 - (-3)^2$

• -4

13. • $6x + 2 = x - 5$

• $5x = -7$

• $x = -\frac{7}{5}$

14. • $2a^{-3}$ or $\frac{2}{a^3}$

• $\sqrt{x} + 3\sqrt{2} = 4\sqrt{2}$

• $\sqrt{x} = \sqrt{2}$

• $x = 2$

15. • $4 \cdot 60$

• $2 \cdot 79$

16. • $\frac{2}{5} \times \frac{10}{11}$

• $\frac{4}{11}$ or equivalent

17. • $7s + 4 = 2t$

• $7s = 2t - 4$

• $s = \frac{2t-4}{7}$

18. • $120\% = 900$

• $20\% = 150$ or similar

• 750

19. • 6

• $4\sqrt{2}$

• $\frac{6}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

• $\frac{3\sqrt{2}}{4}$

10.
- 0·091

20.
- 28.2
- 27.11

21.
- $4\frac{2}{6}-1\frac{3}{6}$
- $\frac{17}{6}$

22.
- $(x-2y)(x+2y)$
- $2x^2+7x-4$
- $3x^{\frac{3}{2}}$ or $x^{-\frac{3}{2}}$
- $3x^{\frac{3}{2}}$ or $x^{-\frac{3}{2}}$

23.
- $BC^2=8^2-4^2$
- $\sqrt{48}$
- $4\sqrt{3}$

24.
- $130\%=260$
- $100\%=\frac{260}{1.3}$
- 200

25.
- $kd^2=20f$
- $d^2=\frac{20f}{k}$
- $d=\sqrt{\frac{20f}{k}}$

26.
- 5.8

27.
- $5(x^2-9)$
- $5(x-3)(x+3)$

28.
- $H^2=\frac{W}{B}$
- $H=\sqrt{\frac{W}{B}}$

29.
- $y=-2x+18$

30.
- $\frac{\ldots}{p(p+5)}$ or $\frac{\ldots}{p^2+5p}$
- $\frac{3p+5}{\ldots}$

31.
- $m^{\frac{7}{2}}$

32.
- $AC^2+\left(\sqrt{32}\right)^2=\left(\sqrt{50}\right)^2$
- $\sqrt{18}$
- $3\sqrt{2}$

33.
- $a=5$
- $b=-7$

34.
- 74·4
- 80·44

35.
- $\times\frac{3}{5}$
- $\frac{19}{10}$ or equivalent

36.
- $\frac{3P+8}{2}$

37.
- $4x^2+6x+6x+9$
- $-3x^2+18$
- $x^2+12x+27$

38.
- $f=\frac{4}{5}d+2$

39.
- $a-2a^{\frac{1}{2}}$

40.
- $b=2, c=3$

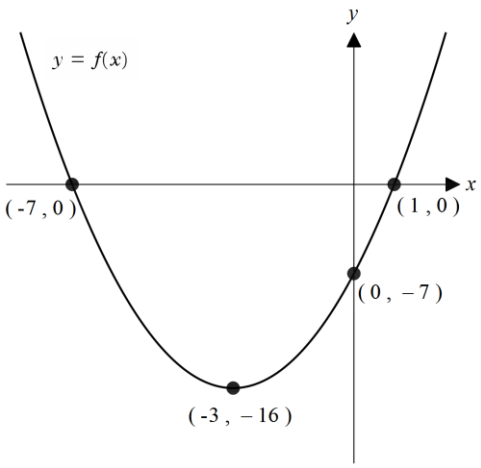
41.
- $(x+2)^2+4$

42.
- $\frac{15}{16}$

43.
- $(x-5)(x+3)$
- $\frac{2}{x+3}$

44.
- $(x-1)^2+2$
- Min value = 2

45.
- ^{1} –5
- ^{2} 1
- ^{1} $x=5$
- ^{1} (0, ?) and (10, ?)
- ^{2} $y=(0-5)^2+1$
- ^{3} P (0, 26), Q (10, 26)

46.
- $(-3,-16)$
- $(0,-7)$
- Multiply out brackets and simplify
- $(-7,0),(1,0)$
- 

47.
- $a=-1$
- $b=\frac{2}{3}$

48.
- $(2,\ 0,\ 3)$

49.
- $4\sqrt{5}$

50.
- $\left(x+\frac{3}{2}\right)^2-\frac{1}{4}$

51.
- $\frac{4\sqrt{7}}{7}$

Section B

Operational Skills – Calculator Allowed (OC)

1. A snail crawls 3 kilometres in 16 days.

What is the average speed of the snail in metres per second?

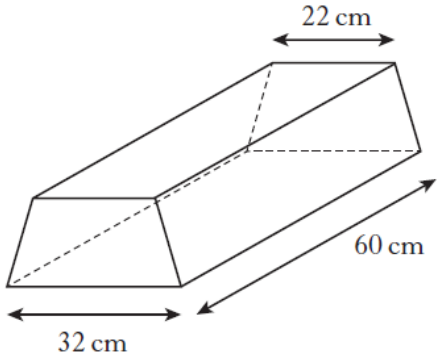
Give your answer **in scientific notation correct to 2 significant figures.**

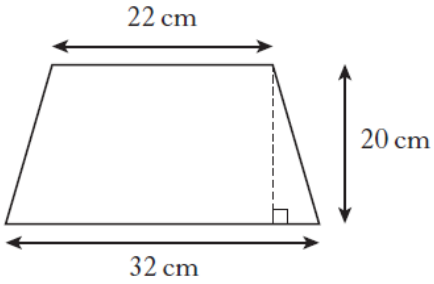
2. Solve the equation

$$2x^2 + 7x - 3 = 0.$$

Give your answers **correct to 1 decimal place.**

3. A concrete block is in the shape of a prism.





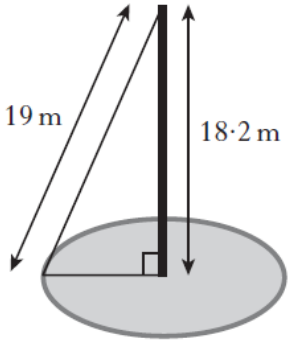
The cross section of the prism is a trapezium with dimensions as shown.

(*a*) Calculate the area of the cross section.

(*b*) Calculate the volume of the concrete block.

4. A mobile phone mast, 18·2 metres high, stands vertically in the centre of a circle.

It is supported by a wire rope, 19 metres long, attached to the ground at a point on the circumference of the circle, as shown.



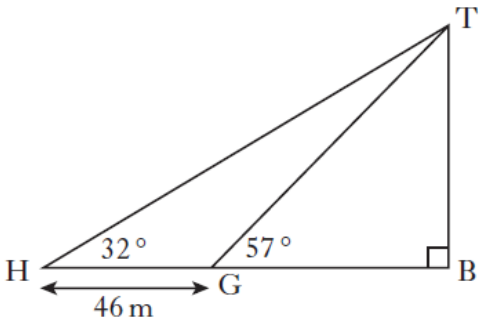
Calculate the circumference of the circle.

5. In triangle THB:

- angle TBH = 90°
- angle THB = 32°.

G is a point on HB.

- angle TGB = 57°
- GH = 46 metres.



Calculate the length of TB.

6. There are 2·69 million vehicles in Scotland.

It is estimated that this number will increase at a rate of 4% each year.

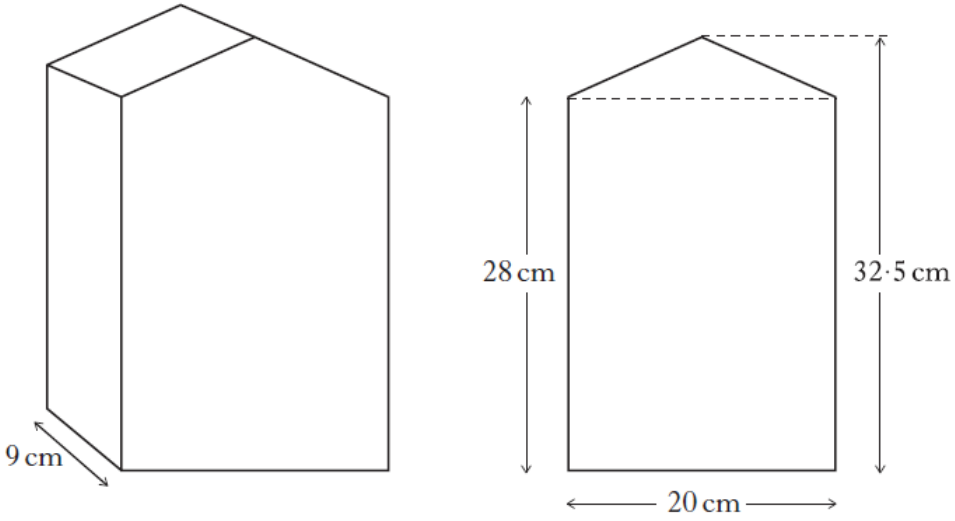
If this estimate is correct, how many vehicles will there be in 3 years' time?

Give your answer **correct to 3 significant figures**.

7. A container for oil is in the shape of a prism.

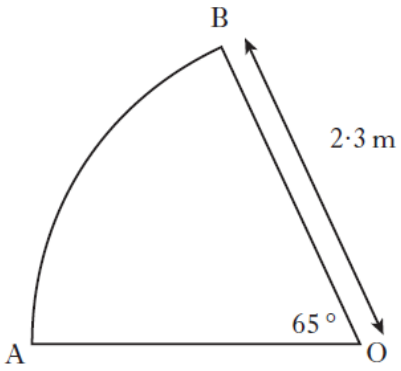
The width of the container is 9 centimetres.

The uniform cross section of the container consists of a rectangle and a triangle with dimensions as shown.



Calculate the volume of the container, **correct to the nearest litre.**

8. A sector of a circle, centre O, is shown below.



The radius of the circle is 2·3 metres.

Angle AOB is 65 °.

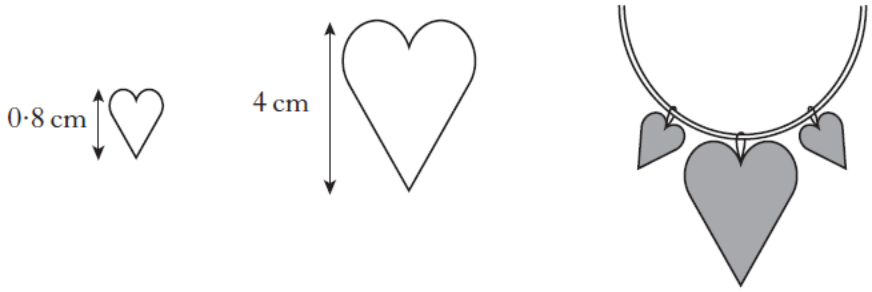
Find the length of the arc AB.

9. The price for Paul's summer holiday is £894·40.

The price includes a 4% booking fee.

What is the price of his holiday without the booking fee?

10. A necklace is made of beads which are mathematically similar.



The height of the smaller bead is 0·8 centimetres and its area is 0·6 square centimetres.

The height of the larger bead is 4 centimetres.

Find the area of the larger bead.

11. Olga normally runs a total distance of 28 miles per week.

She decides to increase her distance by 10% a week for the next four weeks.

How many miles will she run in the fourth week?

12. Expand and simplify

$$(3x + 1)(x^2 - 5x + 4).$$

13. Solve the equation

$$2x^2 + 3x - 7 = 0.$$

Give your answers **correct to 2 significant figures**.

14. A car is valued at £3780.

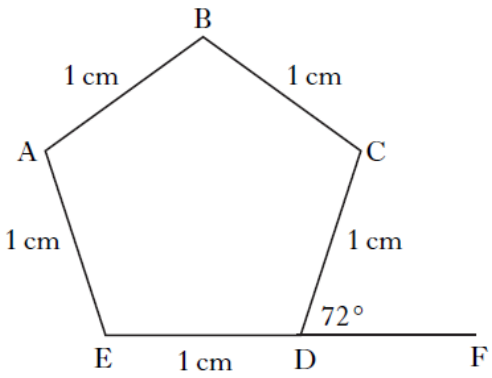
This is 16% less than last year's value.

What was the value of the car last year?

15. ABCDE is a regular pentagon with each side 1 centimetre.

Angle CDF is 72°.

EDF is a straight line.



(*a*) Write down the size of angle ABC.

(*b*) Calculate the length of AC.

16. Tom and Samia are paid the same hourly rate.

Harry is paid $\frac{1}{3}$ more per hour than Tom.

Tom worked 15 hours, Samia worked 8 hours and Harry worked 12 hours.

They were paid a total of £429.

How much was Tom paid?

17. It is estimated that an iceberg weighs 84 000 tonnes.

As the iceberg moves into warmer water, its weight decreases by 25% each day.

What will the iceberg weigh after 3 days in the warmer water?

Give your answer **correct to three significant figures**.

18. Expand fully and simplify

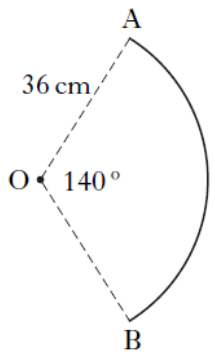
$$x(x-1)^2.$$

19. Use the quadratic formula to solve the equation,

$$3x^2 + 5x - 7 = 0.$$

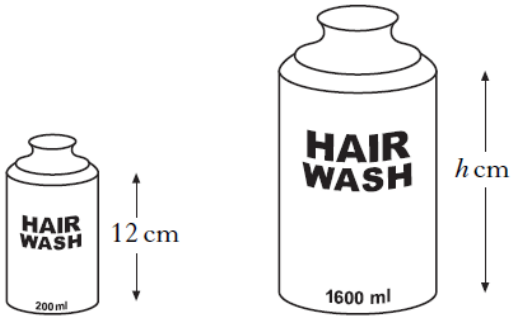
Give your answers correct to **1 decimal place**.

20. A circle, centre O, has radius 36 centimetres.
Part of this circle is shown.
Angle AOB = 140°.



Calculate the length of arc AB.

21. Shampoo is available in travel size and salon size bottles.
The bottles are mathematically similar.



travel salon

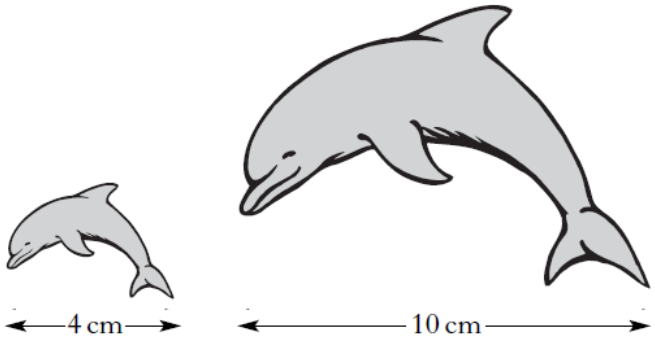
The travel size contains 200 millilitres and is 12 centimetres in height.
The salon size contains 1600 millilitres.
Calculate the height of the salon size bottle.

22. One atom of gold weighs $3{\cdot}27 \times 10^{-22}$ grams.
How many atoms will there be in one kilogram of gold?
Give your answer **in scientific notation correct to 2 significant figures**.

23. Solve the quadratic equation $x^2 - 4x - 6 = 0$.
Give your answers **correct to 1 decimal place**.

24. Two fridge magnets are mathematically similar.

One magnet is 4 centimetres long and the other is 10 centimetres long.



The area of the smaller magnet is 18 square centimetres.

Calculate the area of the larger magnet.

25. A local council recycles 42 000 tonnes of waste a year.

The council aims to increase the amount of waste recycled by 8% each year.

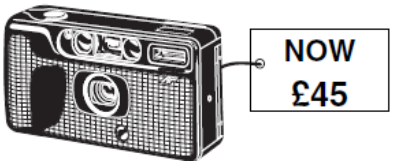
How much waste does it expect to recycle in 3 years time?

Give your answer **to three significant figures**.

26. In a sale, all cameras are reduced by 20%.

A camera now costs £45.

Calculate the **original** cost of the camera.



27. Alistair buys an antique chair for £600.

It is expected to increase in value at the rate of 4·5% each year.

How much is it expected to be worth in 3 years?

28. Solve the equation

$$3x^2 - 2x - 10 = 0.$$

Give your answer **correct to 2 significant figures**.

29. Solve the inequality

$$\frac{x}{4} - \frac{1}{2} < 5.$$

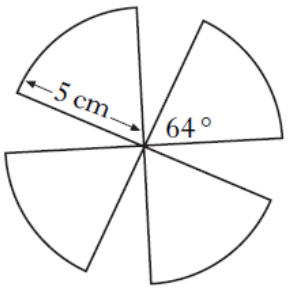
30. Mark takes some friends out for a meal.

The restaurant adds a 10% service charge to the price of the meal.

The **total** bill is £148·50.

What was the price of the meal?

31. A fan has four identical plastic blades.



Each blade is a sector of a circle of radius 5 centimetres.

The angle at the centre of each sector is 64°.

Calculate the **total** area of plastic required to make the blades.

32. Solve **algebraically** the equation

$$5\cos x° + 4 = 0, \qquad 0 \leq x < 360.$$

[END OF SECTION B]

Section B - Answers

1. •[1] $\frac{3km}{16\text{ days}}$

•[2] $\frac{3000}{16\times24\times60\times60}$

•[3] 0·00217

•[4] $2{\cdot}2\times10^{-3}$

2. •[1] $\frac{-7\pm\sqrt{7^2-4\times2\times-3}}{4}$

•[2] 73

•[3] 0·385, −3·885

•[4] 0·4, −3·9

3. •[1] $2\times\frac{1}{2}\times5\times20+(22\times20)$

or

$\frac{1}{2}\times20\times(22+32)$

•[2] 100 + 440
or
10 × 54

•[3] 540

•[1] 32 400

4. •[1] $r^2=19^2-18{\cdot}2^2$

•[2] 5·455

•[3] 34·3

5. •[1] $\frac{TG}{\sin32^\circ}=\frac{46}{\sin25^\circ}$

•[2] TG = 57·679…

•[3] $\sin57^\circ=\frac{TB}{57{\cdot}679...}$

•[4] TB = 48·37…

6. • 1·04

• $1{\cdot}04^3$

• 3 025 884

• 3·03 million

7. • $(28\times20)+\left(\frac{1}{2}\times20\times4{\cdot}5\right)$ (605)

• $\left[(28\times20)+\left(\frac{1}{2}\times20\times4{\cdot}5\right)\right]\times9$

• 5445

• 5

8. • $\frac{65}{360}$ or $\frac{360}{65}$

• $\frac{65}{360}\times\pi\times4{\cdot}6$ or $\pi\times4{\cdot}6\div\frac{360}{65}$

• 2·609

9. • 104% = 894·40

• 1% = $\frac{894{\cdot}4}{104}$ or similar

• 860

10. • $\frac{4}{0{\cdot}8}$

• $\left(\frac{4}{0{\cdot}8}\right)^2=25$

• 15

11. • 1·1

• $1{\cdot}1^4$

• 40·9948
or
40·995
or
40·99

12. • any 3 correct terms

• $3x^3-15x^2+12x+x^2-5x+4$

• $3x^3-14x^2+7x+4$

13. • substitution into quadratic formula

• $\sqrt{65}$

• -2·765, 1·265

• - 2·8, 1·3

14.
- $84\% = £3780$
- $100\% = \frac{3780}{0.84}$
- 4500

15.
- 108°
- 1·62

16.
- $12 + \frac{1}{3}(12)$ or $x + \frac{1}{3}x$
- $15 + 8 + 16 = 39$
  Or
- $15x + 8x + 12\left(\frac{4}{3}x\right) = 39x$
- $\frac{429}{39} \times 15 = £165$

17.
- 0·75
- $0 \cdot 75^3$
- 35 437·5
- 35 400

18.
- $x^3 - 2x^2 + x$

19.
- substitution into quadratic formula
- $\sqrt{109}$
- −2·573, 0·907
- −2·6, 0·9

20.
- $\frac{140}{360}$
- $\frac{140}{360} \times \pi \times 72$
- 87·96

21.
- 8 or equivalent
- $\sqrt[3]{8}$
- 24

22.
- $\frac{1000}{3.27 \times 10^{-22}}$
- 3.058×10^{24}
- 3.1×10^{24}

23.
- substitution into quadratic formula
- $\sqrt{40}$
- -1.162, 5.162
- - 1.2, 5.2

24.
- $\frac{10}{4}$ or $\frac{4}{10}$
- $\left(\frac{5}{2}\right)^2$ or $\left(\frac{2}{5}\right)^2$
- 112.5

25.
- 1.08
- 1.08^3
- 52 907.90
- 52 900

26.
- 80% = 45
- $100\% = \frac{45}{0.8}$
- 56.25

27.
- 1·045
- $1 \cdot 045^3$
- £684·70

28.
- substitution into quadratic formula
- $\sqrt{124}$
- 2·19, -1·52
- 2·2, -1·5

29.

- $x - 2 < 20$ or $\frac{x}{4} < 5\frac{1}{2}$
- $x < 22$

30.

- $110\% = £148{\cdot}50$
- $1\% \quad = £1{\cdot}35$
- £135

31.

- $\frac{64}{360}$
- $\frac{64}{360} \times \pi \times 5^2$
- $13{\cdot}96 \times 4 = 55{\cdot}84$

32.

- $\cos x^\circ = -\frac{4}{5}$
- $143{\cdot}1^\circ$
- $216{\cdot}9^\circ$

Section C

Reasoning Skills – No Calculator (RNC)

1. A group of people attended a course to help them stop smoking.

The following table shows the statistics before and after the course.

	Mean number of cigarettes smoked per person per day	***Standard deviation***
Before	20·8	8·5
After	9·6	12·0

Make **two** valid comments about these results.

2. Joan buys gold and silver charms to make bracelets.

2 gold charms and 5 silver charms cost £125.

(*a*) Let *g* pounds be the cost of one gold charm and *s* pounds be the cost of one silver charm.

Write down an equation in terms of *g* and *s* to illustrate the above information.

4 gold charms and 3 silver charms cost £145.

(*b*) Write down another equation in terms of *g* and *s* to illustrate this information.

(*c*) Hence calculate the cost of each type of charm.

3. Quick-Smile photographers charge the following rates:

- 50p per photograph for the first 12 photographs printed
- 35p per photograph for any further photographs printed
- £4·25 for a CD of the photographs.

(*a*) How much will it cost to have 16 photographs printed plus a CD?

(*b*) Find a formula for C, the cost in pounds, of having *x* photographs printed (where *x* is greater than 12) plus a CD.

4 Jenny is doing calculations using consecutive numbers.

She notices a pattern which always gives an answer of 1.

Using 2, 3, 4 gives $3^2 - 2\times4 = 1$.

3, 4, 5 gives $4^2 - 3\times5 = 1$.

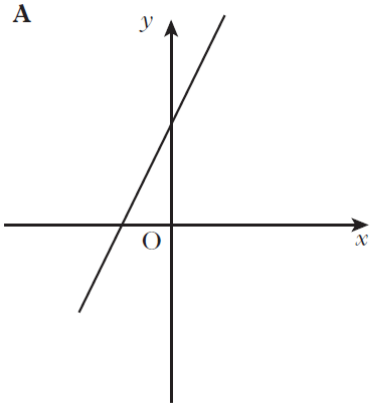
4, 5, 6 gives $5^2 - 4\times6 = 1$.

(*a*) Using 8, 9, 10, write down a similar pattern.

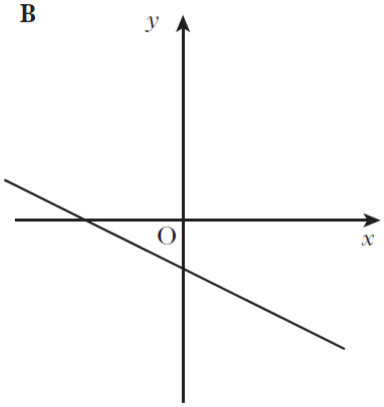
(*b*) Using n, (n+1), (n+2), show that the answer is 1 for any three consecutive numbers.

5. Four straight line graphs are shown below.

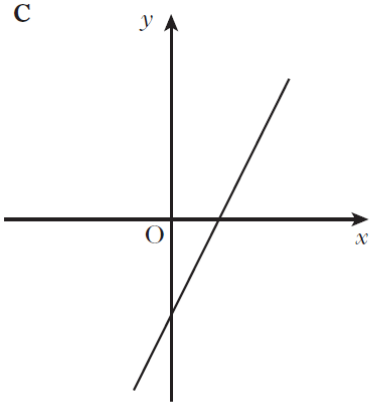
A



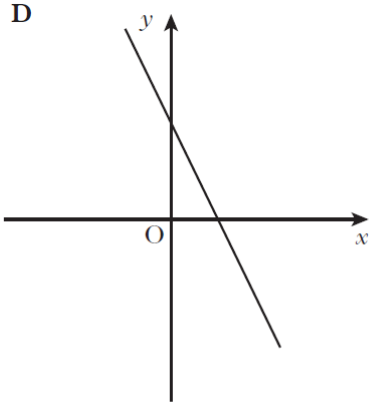
B



C



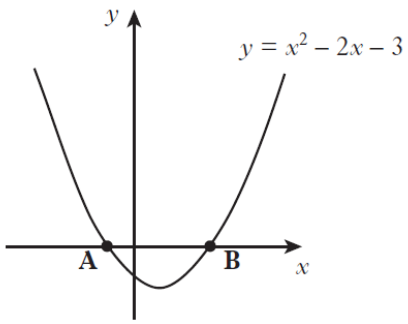
D



Which one of these above could represent the line with equation $2x + y = 3$?

Give two reasons to justify your answer.

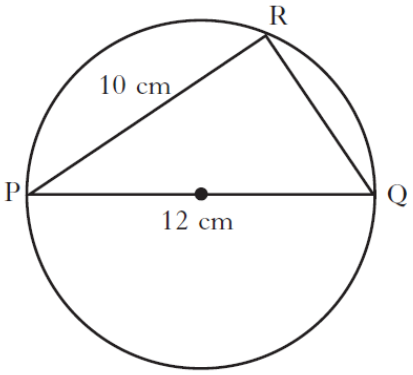
6. The parabola with equation $y = x^2 - 2x - 3$ cuts the x-axis at the points A and B as shown in the diagram.



(*a*) Find the coordinates of A and B.

(*b*) Write down the equation of the axis of symmetry of $y = x^2 - 2x - 3$.

7. In the diagram,



- PQ is the diameter of the circle
- PQ = 12 centimetres
- PR = 10 centimetres.

Calculate the length of QR.

Give your answer as a surd in its simplest form.

8. Mike is practising his penalty kicks.

Last week, Mike scored 18 out of 30.

This week, he scored 16 out of 25.

Has his scoring rate improved?

Give a reason for your answer.

9. Given $2x^2 - 2x - 1 = 0$, show that

$$x = \frac{1 \pm \sqrt{3}}{2}$$

10. Each day, Marissa drives 40 kilometres to work.

(*a*) On Monday, she drives at a speed of x kilometres per hour.

Find the time taken, in terms of x, for her journey.

(*b*) On Tuesday, she drives 5 kilometres per hour **faster**.

Find the time taken, in terms of x, for this journey.

(*c*) Hence find an expression, in terms of x, for the difference in times of the two journeys.

Give this expression **in its simplest form**.

11. (*a*) Evaluate $(2^3)^2$.

(*b*) Hence find n, when $(2^3)^n = \frac{1}{64}$.

12. The sum of consecutive even numbers can be calculated using the following number pattern:

$$2 + 4 + 6 = 3 \times 4 = 12$$
$$2 + 4 + 6 + 8 = 4 \times 5 = 20$$
$$2 + 4 + 6 + 8 + 10 = 5 \times 6 = 30$$

(*a*) Calculate $2 + 4 + \cdots + 20$.

(*b*) Write down an expression for $2 + 4 + \cdots + n$.

(*c*) Hence or otherwise calculate $10 + 12 + \cdots + 100$.

13. Jamie is going to bake cakes for a party.

He needs $\frac{2}{5}$ of a block of butter for 1 cake.

He has 7 blocks of butter.

How many cakes can Jamie bake?

14. Brian, Molly and their four children visit Waterworld.

The total cost of their tickets is £56.

Let a pounds be the cost of an adult's ticket and c pounds the cost of a child's ticket.

Write down an equation in terms of a and c to illustrate this information.

Sarah and her three children visit Waterworld.

The total cost of their tickets is £36.

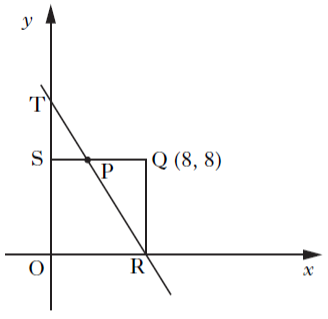
Write down another equation in terms of a and c to illustrate this information.

(i) Calculate the cost of a child's ticket.

(ii) Calculate the cost of an adult's ticket.

15. A square, OSQR, is shown below.

Q is the point (8, 8).



The straight line TR cuts the y-axis at T (0, 12) and the x-axis at R.

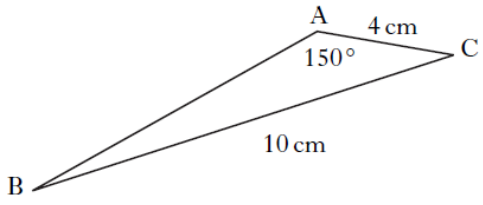
(a) Find the equation of the line TR.

The line TR also cuts SQ at P.

(b) Find the coordinates of P.

16. In triangle ABC

- AC = 4 centimetres
- BC = 10 centimetres
- angle BAC = 150°



Given that $\sin 30° = \frac{1}{2}$, show that $\sin B = \frac{1}{5}$.

17. The sums, S_2, S_3 and S_4 of the first 2, 3 and 4 natural numbers are given by:

$$S_2 = 1 + 2 \qquad = \frac{1}{2}(2 \times 3) = 3$$

$$S_3 = 1 + 2 + 3 \qquad = \frac{1}{2}(3 \times 4) = 6$$

$$S_4 = 1 + 2 + 3 + 4 \qquad = \frac{1}{2}(4 \times 5) = 10$$

(*a*) Find S_{10}, the sum of the first 10 natural numbers.

(*b*) Write down the formula for the sum, S_n, of the first n natural numbers.

18. Two functions are given below.

$$f(x) = x^2 - 4x$$

$$g(x) = 2x + 7$$

(*a*) If $f(x) = g(x)$, show that $x^2 - 6x - 7 = 0$.

(*b*) Hence find **algebraically** the values of x for which $f(x) = g(x)$.

19. A straight line has equation $y = mx + c$, where m and c are constants.

(*a*) The point (2, 7) lies on this line.

Write down an equation in m and c to illustrate this information.

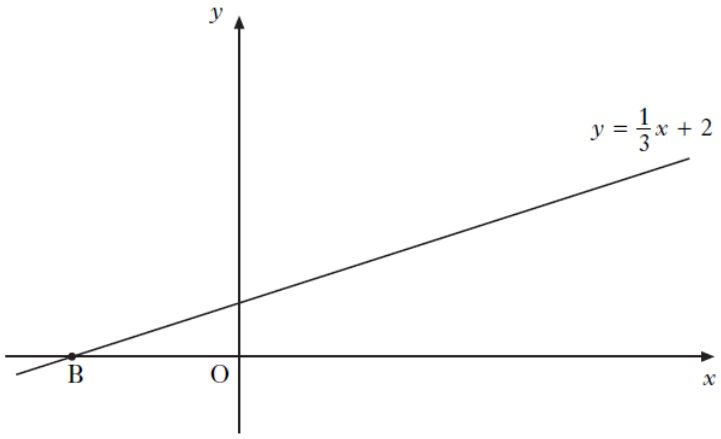
(*b*) A second point (4, 17) also lies on this line.

Write down another equation in m and c to illustrate this information.

(*c*) Hence calculate the values of m and c.

(*d*) Write down the gradient of this line.

20. Part of the graph of the straight line with equation $y = \frac{1}{3}x + 2$, is shown below.



(*a*) Find the coordinates of the point B.

(*b*) For what values of x is $y < 0$?

21. A number pattern is shown below.

$$1^3 = \frac{1^2 \times 2^2}{4}$$

$$1^3 + 2^3 = \frac{2^2 \times 3^2}{4}$$

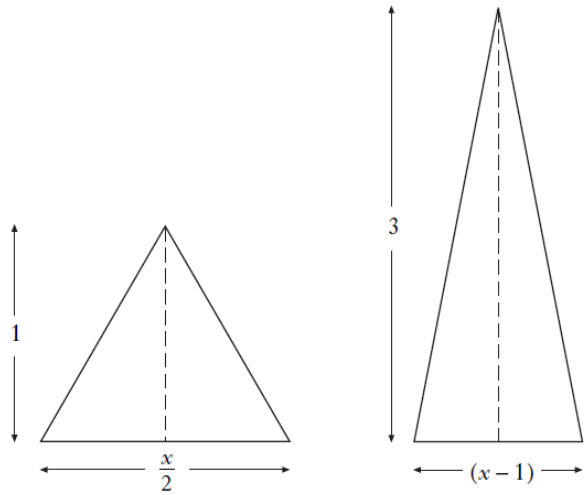
$$1^3 + 2^3 + 3^3 = \frac{3^2 \times 4^2}{4}$$

(*a*) Write down a similar expression for $1^3 + 2^3 + 3^3 + 4^3 + 5^3$.

(*b*) Write down a similar expression for $1^3 + 2^3 + 3^3 + \ldots + n^3$.

(*c*) Hence **evaluate** $1^3 + 2^3 + 3^3 + \ldots + 9^3$.

22. Two triangles have dimensions as shown.



The triangles are equal in area.

Calculate the value of x.

23. Given that

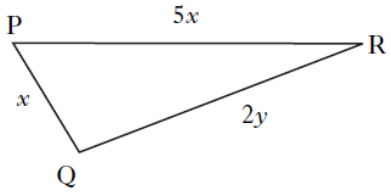
$$f(x) = x^2 + 3,$$

(*a*) evaluate $f(-4)$

(*b*) find t when $f(t) = 52$.

24. In triangle PQR:

- PQ = x centimetres
- PR = $5x$ centimetres
- QR = $2y$ centimetres.



(*a*) The perimeter of the triangle is 42 centimetres.

Write down an equation in x and y to illustrate this information.

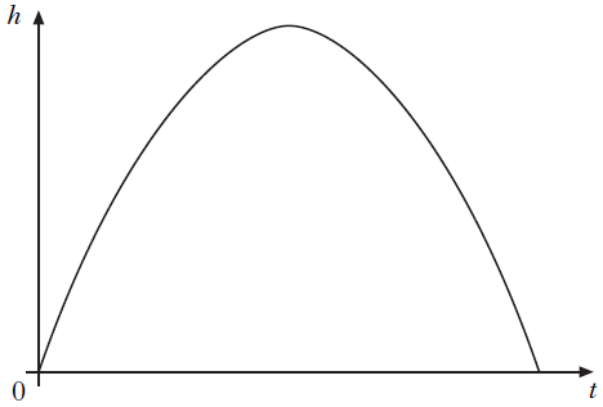
(*b*) PR is 2 centimetres longer than QR.

Write down another equation in x and y to illustrate this information.

(*c*) Hence calculate the values of x and y.

25. The diagram below shows the path of a rocket which is fired into the air.
The height, h metres, of the rocket after t seconds is given by

$$h(t) = -2t(t - 14).$$

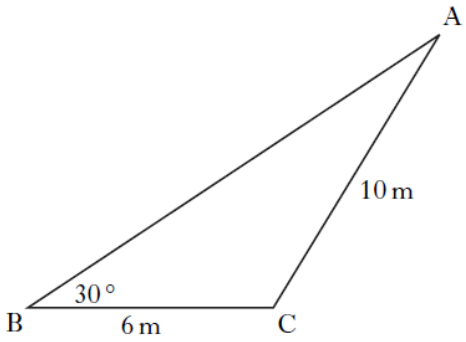


(*a*) For how many seconds is the rocket in flight?

(*b*) What is the maximum height reached by the rocket?

26. In triangle ABC:

- BC = 6 metres
- AC = 10 metres
- angle ABC = 30 °.



Given that sin 30 ° = 0·5, show that sin A = 0·3.

27. Jane enters a two-part race.

(*a*) She cycles for 2 hours at a speed of $(x + 8)$ kilometres per hour.
Write down an expression in x for the distance cycled.

(*b*) She then runs for 30 minutes at a speed of x kilometres per hour.
Write down an expression in x for the distance run.

(*c*) The **total** distance of the race is 46 kilometres.
Calculate Jane's **running** speed.

28. The 4th term of each number pattern below is the **mean** of the previous three terms.

(*a*) When the first three terms are 1, 6, and 8, calculate the 4th term.

(*b*) When the first three terms are x, $(x + 7)$ and $(x + 11)$, calculate the 4th term.

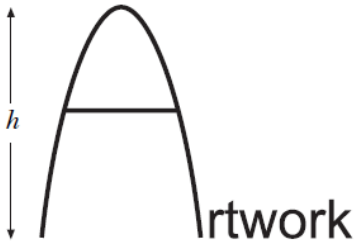
(*c*) When the first, second and fourth terms are

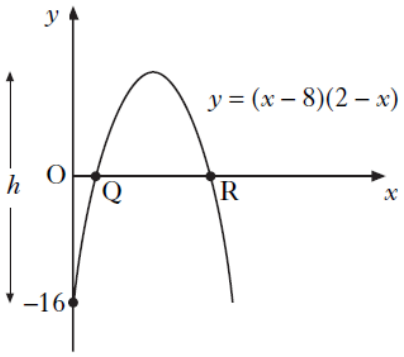
$$-2x, \qquad (x + 5), \qquad \_\_\_\_, \qquad (2x + 4),$$

calculate the 3rd term.

29. The curved part of the letter A in the *Artwork* logo is in the shape of a parabola.

The equation of this parabola is $y = (x - 8)(2 - x)$.





(*a*) Write down the coordinates of Q and R.

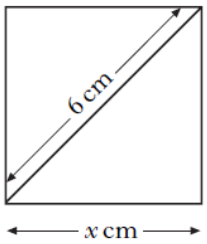
(*b*) Calculate the height, h, of the letter A.

30. A new fraction is obtained by adding x to the numerator and denominator of the fraction $\frac{17}{24}$.

This new fraction is equivalent to $\frac{2}{3}$.

Calculate the value of x.

31. A square of side x centimetres has a diagonal 6 centimetres long.



Calculate the value of x, giving your answer as a surd in its simplest form.

32. A relationship between T and L is given by the formula, $T = \frac{k}{L^3}$ where k is a constant.

When L is doubled, what is the effect on T?

33. (*a*) A cinema has 300 seats which are either standard or deluxe.

Let x be the number of standard seats and y be the number of deluxe seats.

Write down an algebraic expression to illustrate this information.

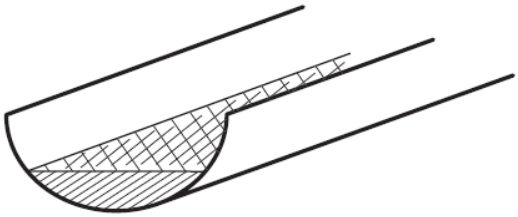
(*b*) A standard seat costs £4 and a deluxe seat costs £6.

When all the seats are sold the ticket sales are £1380.

Write down an algebraic expression to illustrate this information.

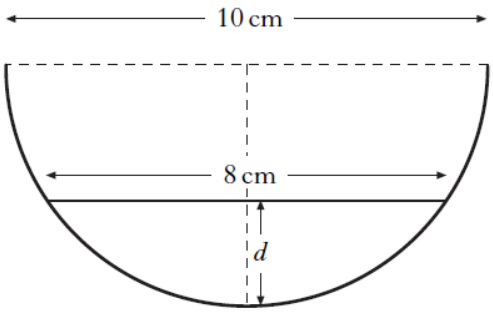
(*c*) How many standard seats and how many deluxe seats are in the cinema?

34. The diagram shows water lying in a length of roof guttering.



The cross-section of the guttering is a semi-circle with diameter 10 centimetres.

The water surface is 8 centimetres wide.



Calculate the depth, d, of water in the guttering.

35. The **sum** S_n of the first n terms of a sequence, is given by the formula

$$S_n = 3^n - 1.$$

(*a*) Find the **sum** of the first 2 terms.

(*b*) When $S_n = 80$, calculate the value of n.

[END OF SECTION C]

Section C - Answers

1. $\bullet^1$ on average fewer cigarettes were smoked per person after the course

$\bullet^2$ the number of cigarettes smoked per person was more varied after the course

2. $\bullet^1$ $2g + 5s = 125$

$\bullet^1$ $4g + 3s = 145$

$\bullet^1$ evidence of scaling

$\bullet^2$ $g = 25$

$\bullet^3$ $s = 15$

3. $\bullet^1$ $y = -2x + 3$

$\bullet^2$ m – negative
c – positive

$\bullet^3$ D

4. $\bullet^1$ either $(12 \times 0{\cdot}5)\ldots\ldots + 4{\cdot}25$
or $(16 - 12) \times 0{\cdot}35$

$\bullet^2$ £11·65

$\bullet^1$ $12 \times 0{\cdot}5$

$\bullet^2$ $(x - 12) \times 0{\cdot}35$

$\bullet^3$ $6 + (x - 12)\ 0{\cdot}35 + 4{\cdot}25$

5. $\bullet^1$ $x^2 - 2x - 3 = 0$

$\bullet^2$ $(x - 3)\ (x + 1) = 0$

$\bullet^3$ $x = -1$ or 3

$\bullet^4$ A(−1,0), B(3,0)

$\bullet^1$ $x = 1$

6. $\bullet^1$ $9^2 - 8 \times 10 = 1$

$\bullet^1$ $(n + 1)^2 - n\ (n + 2)$

$\bullet^2$ $n^2 + 2n + 1 - n^2 - 2n$

$\bullet^3$ 1

7. • $\hat{R} = 90°$ or $PQ^2 = PR^2 + QR^2$ or indication on diagram

• $QR^2 = 12^2 - 10^2$

• $QR = \sqrt{44}$

• $2\sqrt{11}$

8. • $\frac{90}{150}$ or 0·6

• $\frac{96}{150}$ or 0·64

• yes, because $\frac{96}{150} > \frac{90}{150}$

or

$0{\cdot}64 > 0{\cdot}6$

9. • $x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2 \times 2}$

• $\sqrt{12}$

• $2\sqrt{3}$

• $\frac{1 \pm \sqrt{3}}{2}$

10. • $(T =)\frac{40}{x}$

• $(T =)\frac{40}{x+5}$

• $\frac{40}{x} - \frac{40}{x+5}$

• $\frac{\ldots}{x(x+5)} - \frac{\ldots}{x(x+5)}$

• $\frac{200}{x(x+5)}$

11. • 64

• $n = -2$

12. • 110

• $\frac{n}{2} \times \left(\frac{n}{2} + 1\right)$

• 2550

• 2530

13.
- $7 \div \frac{2}{5}$
- $\frac{35}{2}$
- 17

14.
- $2a + 4c = 56$
- $a + 3c = 36$
- $2a + 6c = 72$ or equivalent
- $c = 8$ or $a = 12$
- second value

15.
- $y = -\frac{3}{2}x + 12$
- recognising $y_p = 8$
- $8 = -\frac{3}{2}x + 12$
- $x = \frac{8}{3}$
- $\left(\frac{8}{3}, 8\right)$

16.
- $\frac{10}{\sin 150^\circ} = \frac{4}{\sin B}$
- $10 \sin B = 4 \sin 150^\circ$
- $10 \sin B = 4 \times \frac{1}{2}$
- $\sin B = \frac{1}{5}$

17.
- 55
- $(s_n =) \frac{1}{2} n (n+1)$

18.
- $x^2 - 4x = 2x + 7$
- $x^2 - 6x - 7 = 0$
- $(x + 1)(x - 7)$
- $-1, 7$

19.
- $2m + c = 7$
- $4m + c = 17$
- $2m = 10$ or similar
- 5
- –3
- 5

20.
- $\frac{1}{3}x + 2 = 0$
- $(-6, 0)$
- $x < -6$

21.
- $\frac{5^2 \times 6^2}{4}$
- n and $n + 1$
- $\frac{n^2(n+1)^2}{4}$
- $\frac{9^2(9+1)^2}{4}$
- 2025

22.
- $\frac{1}{2} \times 1 \times \frac{x}{2}$ or $\frac{1}{2} \times 3 \times (x - 1)$
- $\frac{1}{2} \times 1 \times \frac{x}{2} = \frac{1}{2} \times 3 \times (x - 1)$
- $x = 6(x - 1)$
- $x = \frac{6}{5}$

23.
- $(-4)^2 + 3$
- 19
- $t^2 + 3 = 52$
- $t = \pm 7$

24.
- $x + 5x + 2y$
- an equation containing only the terms $5x$, $2y$ and 2
- $x + 5x + 2y = 42$
- $5x - 2y = 2$
- $11x = 44$ or equivalent
- $x = 4$
- $y = 9$

25.
- $-2t(t-14)=0$
- 14
- $(x=)\,7$
- 98

26.
- $\dfrac{10}{\sin 30^\circ}=\dfrac{6}{\sin A}$
- $\sin A=\dfrac{6\sin 30^\circ}{10}$
- 0.3

27.
- $2(x+8)$
- $0.5x$
- $2(x+8)+0.5x=46$
- $2.5x=30$
- $x=12$

28.
- 5
- $x+6$
- $\dfrac{-2x+(x+5)+3^{\text{rd}}\text{ term}}{3}=2x+4$
- $7x+7$

29.
- $(2,0)$
- $(8,0)$
- $x=5$
- $y=9$
- 25 units

30.
- $\dfrac{17+x}{24+x}$
- $\dfrac{17+x}{24+x}=\dfrac{2}{3}$
- $x=-3$

31.
- $x^2+x^2=6^2$
- $x=\sqrt{18}$
- $3\sqrt{2}$

32.
- $(2L)^3$
- $\times\dfrac{1}{8}$ or $\div 8$

33.
- $x+y=300$
- $4x$ and $6y$
- $4x+6y=1380$
- $4x+4y=1200$ or equivalent
- 210
- 90

34.
- recognition of right angle at chord
- correct use of Pythagoras
- 3
- $d=2$

35.
- 8
- $3^n-1=80$
- 4

Section D

Reasoning Skills – Calculator Allowed (RC)

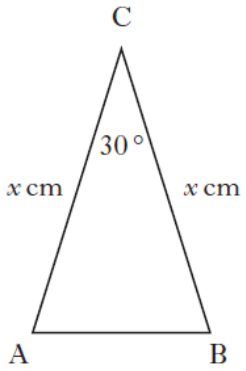
1. Last year, 1296 learner drivers from "Topflight" school of motoring passed their driving test.

This was 72% of those who sat their driving test from Topflight.

How many **failed** their driving test?

2. ABC is an isosceles triangle with angle ACB = 30 °.

AC = BC = x centimetres.



The area of triangle ABC is 9 square centimetres.

Calculate the value of x.

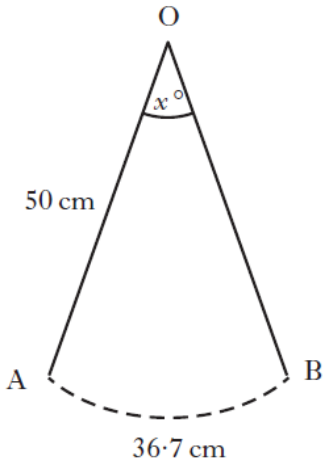
3. Jack weighs 94 kilograms.

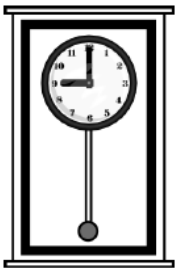
On the 1st of January, he starts a diet which is designed to reduce his weight by 7% per month.

During which month should he achieve his target weight of 73 kilograms?

Show all your working.

4. As the pendulum of a clock swings, its tip moves through an arc of a circle.





The length of the pendulum is 50 centimetres.
The length of the arc is 36·7 centimetres.

Calculate x°, the angle through which the pendulum swings.

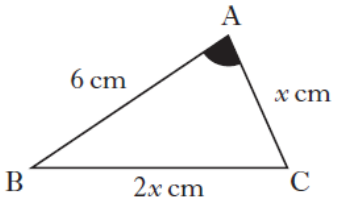
5. A function is given by the formula, $f(x) = 4 \times 2^x$.

(*a*) Evaluate $f(3)$.

(*b*) Given that $f(m) = 4$, find the value of m.

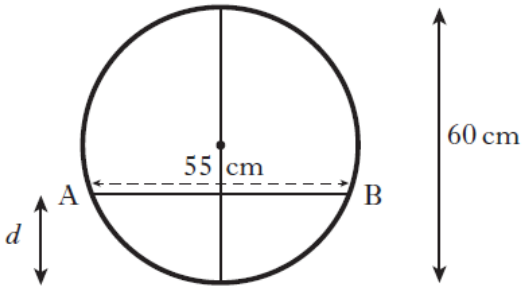
6. In triangle ABC:

- cos A = 0·5
- AB = 6 centimetres
- BC = $2x$ centimetres
- AC = x centimetres.



Show that $x^2 + 2x - 12 = 0$.

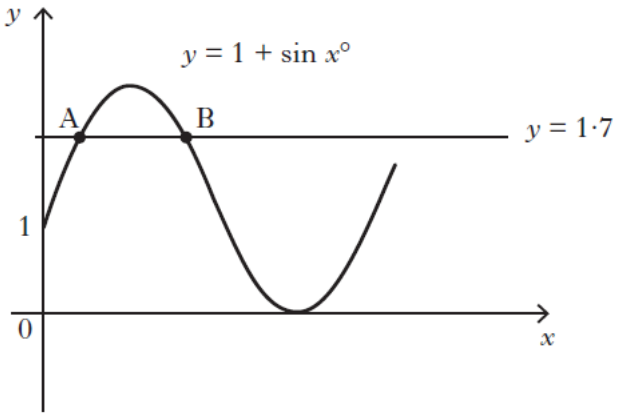
7. Water flows through a horizontal pipe of diameter 60 centimetres.
The surface width, AB, of the water is 55 centimetres.



(*a*) Calculate the depth, d, of the water in the pipe.

(*b*) What other depth of water would give the same surface width?

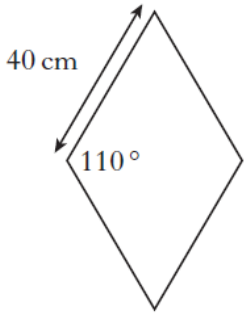
8. Part of the graph of $y = 1 + \sin x°$ is shown in the diagram below.



The line $y = 1{\cdot}7$ is drawn. It cuts the graph of $y = 1 + \sin x°$ at A and B as shown.

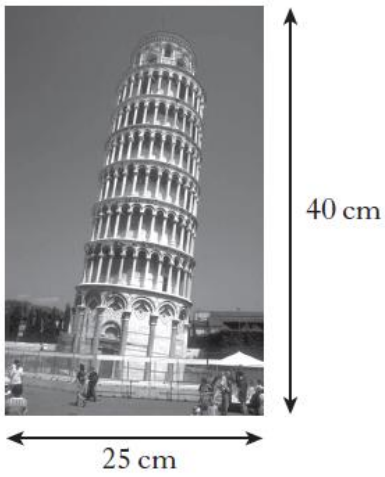
Calculate the x-coordinates of A and B.

9. Paving stones are in the shape of a rhombus.



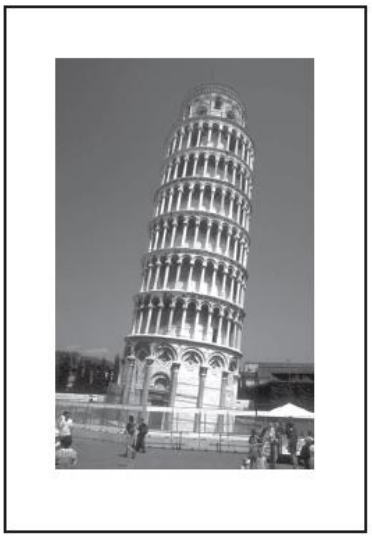
The side of each rhombus is 40 centimetres long.
The obtuse angle is 110°.
Find the area of one paving stone.

10. Asim has a poster which is 25 centimetres wide and 40 centimetres high.

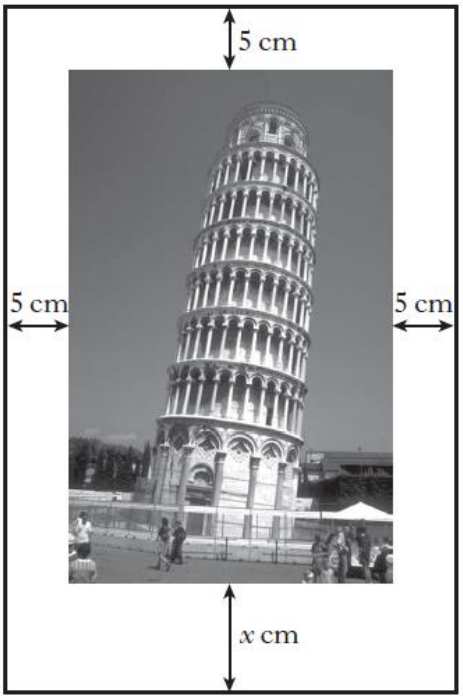


He decides to place it on a white card.

The card and the poster are mathematically similar.



The border is 5 centimetres wide on three sides and x centimetres wide on the fourth side as shown.



Calculate the value of x.

11. Before training, athletes were tested on how many sit-ups they could do in one minute.

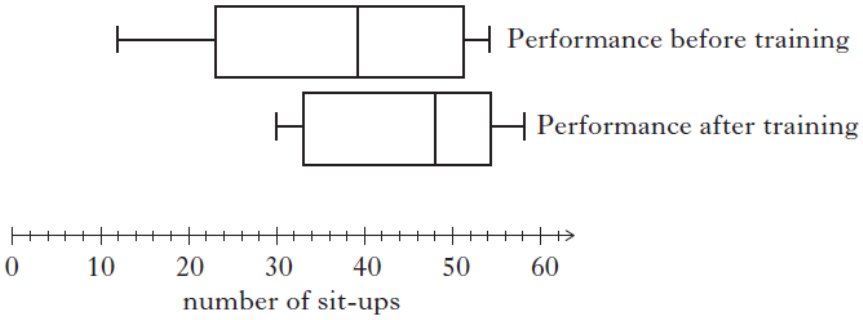
The following information was obtained:

lower quartile (Q_1)	23
median (Q_2)	39
upper quartile (Q_3)	51

(*a*) Calculate the semi-interquartile range.

After training, the athletes were tested again.

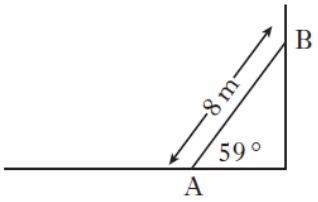
Both sets of data are displayed as boxplots.



(*b*) Make **two** valid statements to compare the performances before and after training.

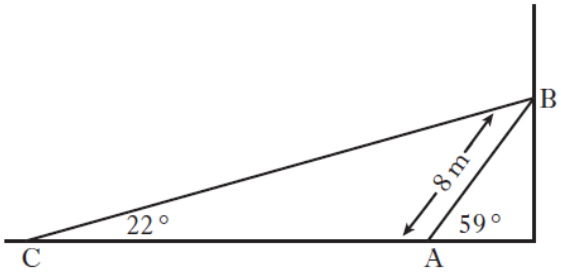
12. A heavy metal beam, AB, rests against a vertical wall as shown.

The length of the beam is 8 metres and it makes an angle of 59° with the ground.



A cable, CB, is fixed to the ground at C and is attached to the top of the beam at B.

The cable makes an angle of 22° with the ground.

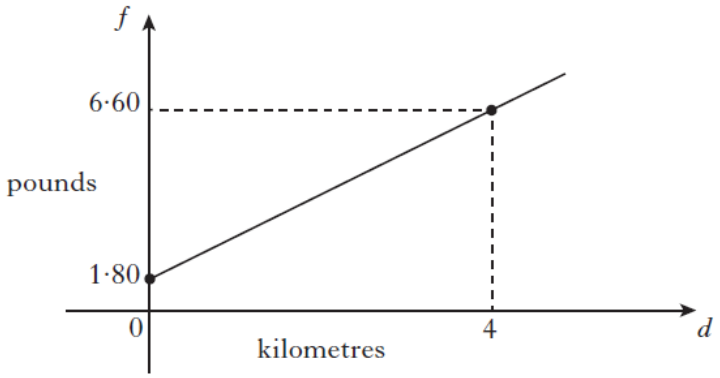


Calculate the length of cable CB.

13. A taxi fare consists of a call-out charge of £1·80 **plus** a fixed cost per kilometre.

A journey of 4 kilometres costs £6·60.

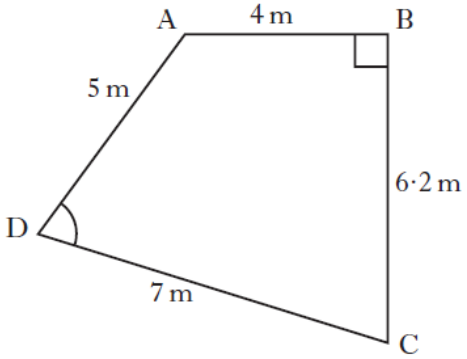
The straight line graph shows the fare, f pounds, for a journey of d kilometres.



(*a*) Find the equation of the straight line.

(*b*) Calculate the fare for a journey of 7 kilometres.

14. Quadrilateral ABCD with angle ABC = 90 ° is shown below.



- AB = 4 metres
- BC = 6·2 metres
- CD = 7 metres
- AD = 5 metres

(*a*) Calculate the length of AC.

(*b*) Calculate the size of angle ADC.

15. $f(x) = 3 \sin x°, \qquad 0 \leq x < 360$

(*a*) Find $f(270)$.

(*b*) $f(t) = 0{\cdot}6$.

Find the two possible values of t.

16. The ratio of sugar to fruit in a particular jam is 5 : 4.

It is decided to:

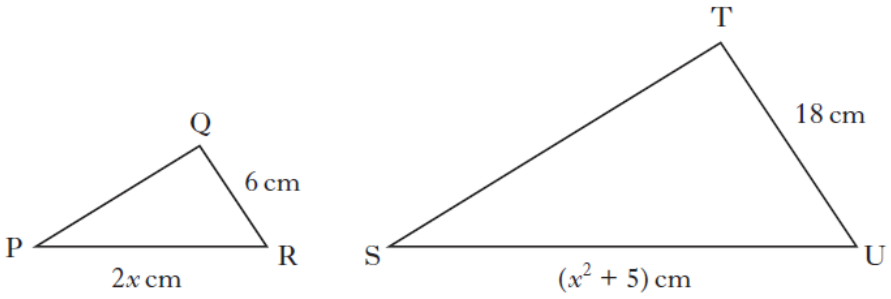
- **decrease** the sugar content by 20%
- **increase** the fruit content by 20%.

Calculate the new ratio of sugar to fruit.

Give your answer in its simplest form.

17. Triangles PQR and STU are mathematically similar.

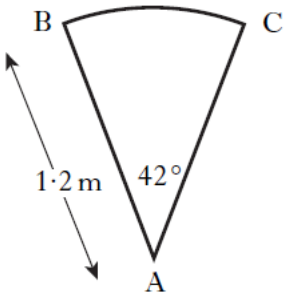
The scale factor is 3 and PR corresponds to SU.



(*a*) Show that $x^2 - 6x + 5 = 0$.

(*b*) Given QR is the shortest side of triangle PQR, find the value of x.

18. A spiral staircase is being designed.



Each step is made from a sector of a circle as shown.

The radius is 1·2 metres.

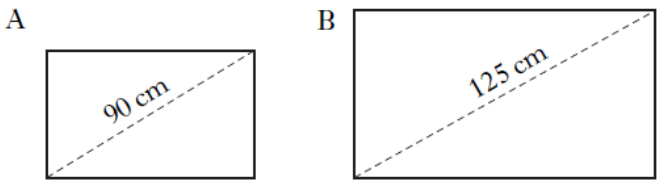
Angle BAC is 42°.

For the staircase to pass safety regulations, the arc BC must be at least 0·9 metres.

Will the staircase pass safety regulations?

19. Two rectangular solar panels, A and B, are mathematically similar.

Panel A has a diagonal of 90 centimetres and an area of 4020 square centimetres.

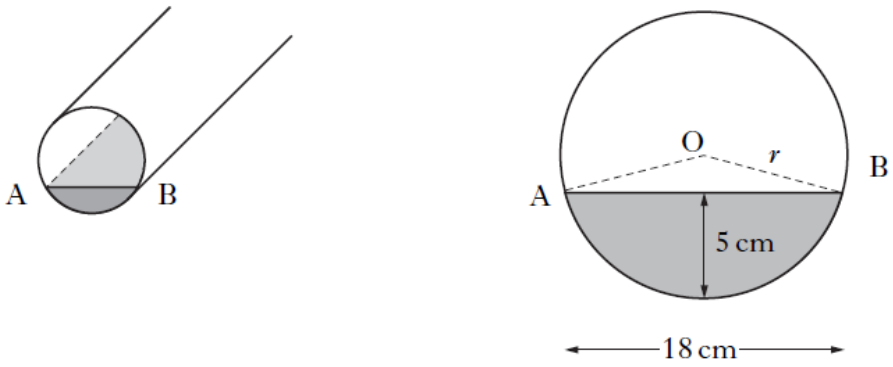


A salesman claims that panel B, with a diagonal of 125 centimetres, will be double the area of panel A.

Is this claim justified?

Show all your working.

20. A pipe has water in it as shown.



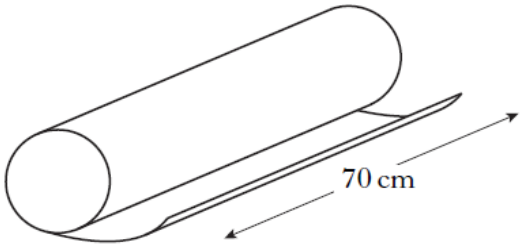
The depth of the water is 5 centimetres.

The width of the water surface, AB, is 18 centimetres.

Calculate r, the radius of the pipe.

21. Paper is wrapped round a cardboard cylinder **exactly** 3 times.

The cylinder is 70 centimetres long.

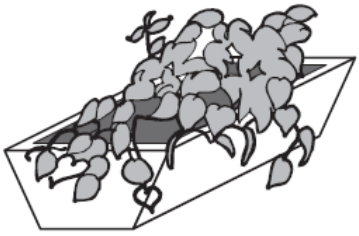


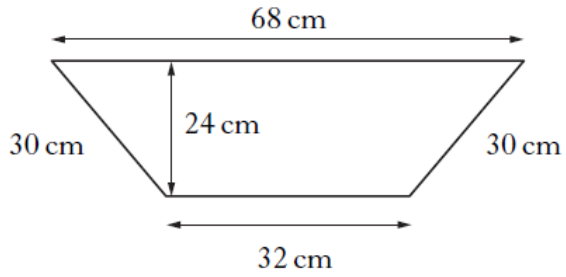
The area of the paper is 3000 square centimetres.

Calculate the diameter of the cylinder.

22. A flower planter is in the shape of a prism.

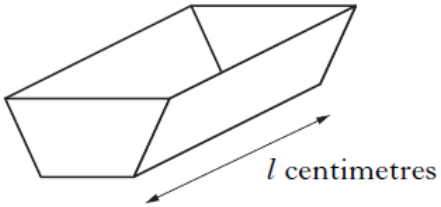
The cross-section is a trapezium with dimensions as shown.





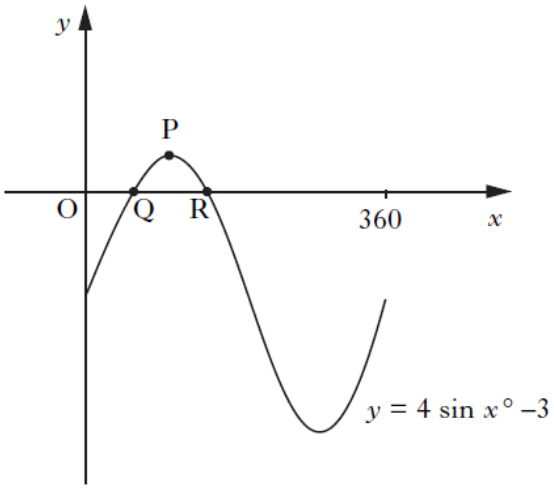
(*a*) Calculate the area of the cross-section of the planter.

(*b*) The volume of the planter is 156 litres.



Calculate the length, *l* centimetres, of the planter.

23. Part of the graph of $y = 4 \sin x° - 3$ is shown below.



The graph cuts the x-axis at Q and R.

P is the maximum turning point.

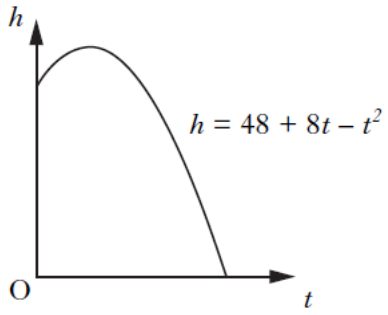
(*a*) Write down the coordinates of P.

(*b*) Calculate the x-coordinates of Q and R.

24. The diagram shows the path of a flare after it is fired.

The height, h metres above sea level, of the flare is given by

$h = 48 + 8t - t^2$ where t is the number of seconds after firing.



Calculate, **algebraically**, the time taken for the flare to enter the sea.

25. A machine is used to put drawing pins into boxes.

A sample of 8 boxes is taken and the number of drawing pins in each is counted.

The results are shown below:

102 102 101 98 99 101 103 102

(*a*) Calculate the mean and standard deviation of this sample.

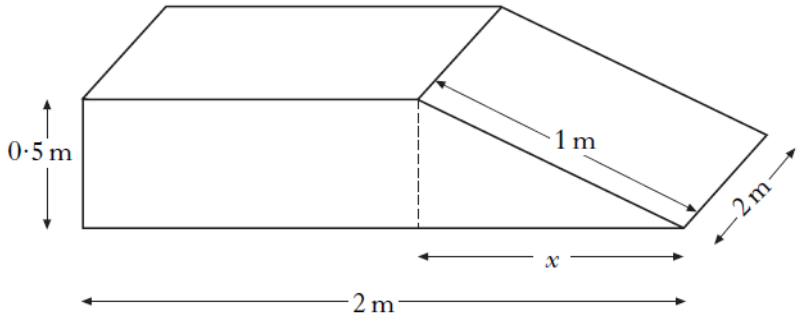
(*b*) A sample of 8 boxes is taken from another machine.

This sample has a mean of 103 and a standard deviation of 2·1.

Write down two valid comparisons between the samples.

26. A concrete ramp is to be built.

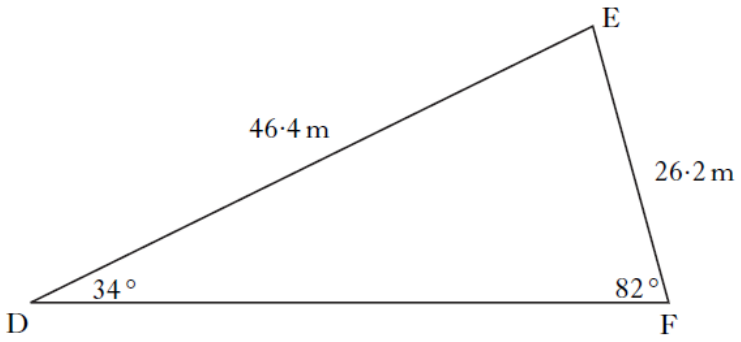
The ramp is in the shape of a cuboid and a triangular prism with dimensions as shown.



(*a*) Calculate the value of x.

(*b*) Calculate the volume of concrete required to build the ramp.

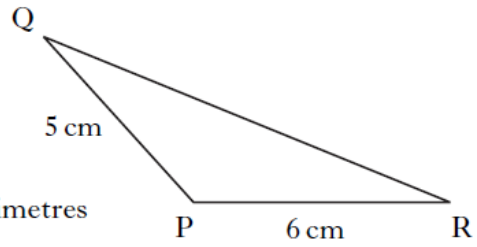
27. As part of their training, footballers run around a triangular circuit DEF.



- ∠EDF = 34°
- ∠DFE = 82°
- DE = 46·4 metres
- EF = 26·2 metres

How many **complete** circuits must they run to cover **at least** 1000 metres?

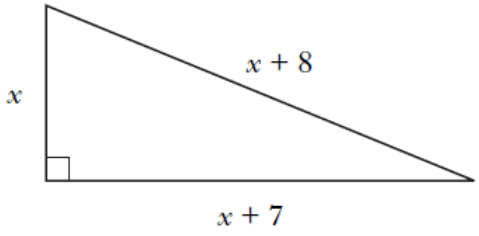
28. In triangle PQR:



- PQ = 5 centimetres
- PR = 6 centimetres
- area of triangle PQR = 12 square centimetres
- angle QPR is **obtuse**.

Calculate the size of angle QPR.

29. A right-angled triangle has dimensions, in centimetres, as shown.



Calculate the value of x.

30. The depth of water, D metres, in a harbour is given by the formula

$$D = 3 + 1{\cdot}75 \sin 30h^{\circ}$$

where h is the number of hours after midnight.

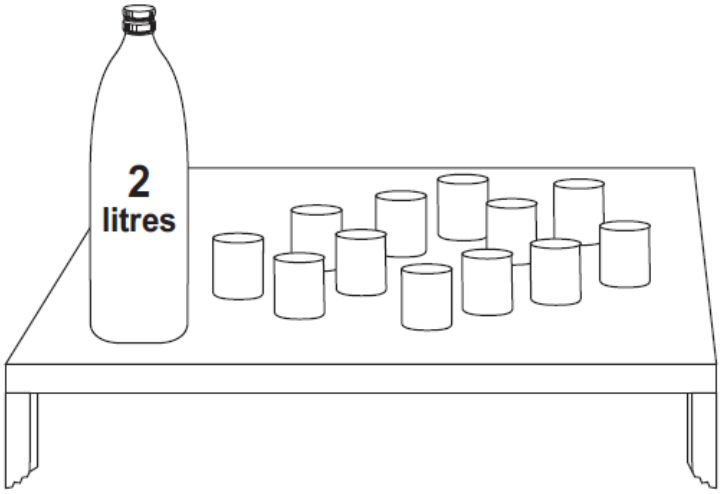
(a) Calculate the depth of water at 5 am.

(b) Calculate the maximum difference in depth of the water in the harbour.

Do not use a trial and improvement method.

31. Lemonade is to be poured from a 2 litre bottle into glasses.

Each glass is in the shape of a cylinder of radius 3 centimetres and height 8 centimetres.



How many full glasses can be poured from the bottle?

32. Tom looked at the cost of 10 different flights to New York.

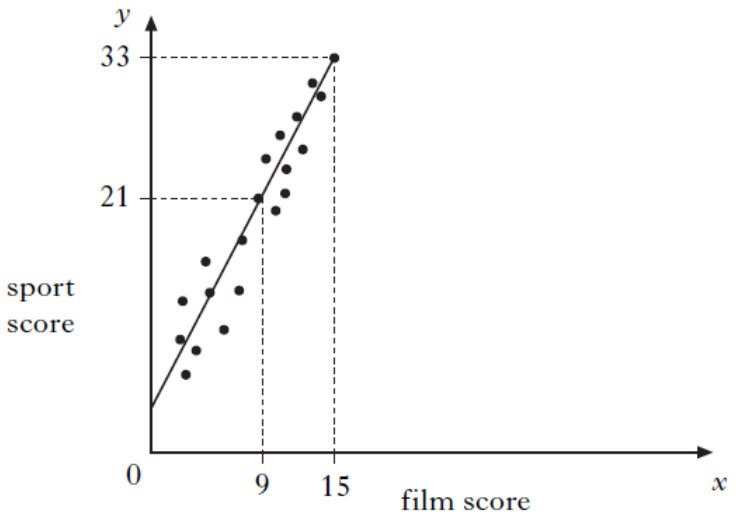
He calculated that the mean cost was £360 and the standard deviation was £74.

A tax of £12 is then added to each flight

Write down the new mean and standard deviation.

33. Teams in a quiz answer questions on film and sport.

This scatter graph shows the scores of some of the teams.



A line of best fit is drawn as shown above.

(*a*) Find the equation of this straight line.

(*b*) Use this equation to estimate the sport score for a team with a film score of 20.

34. A company makes large bags of crisps which contain 90 grams of fat.

The company aims to reduce the fat content of the crisps by 50%.

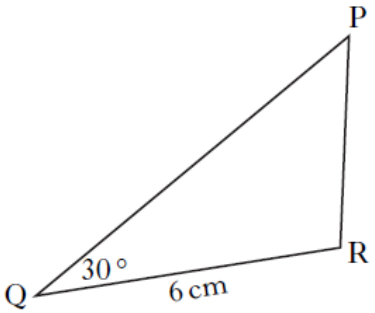
They decide to reduce the fat content by 20% each year.

Will they have achieved their aim by the end of the 3rd year?

Justify your answer.

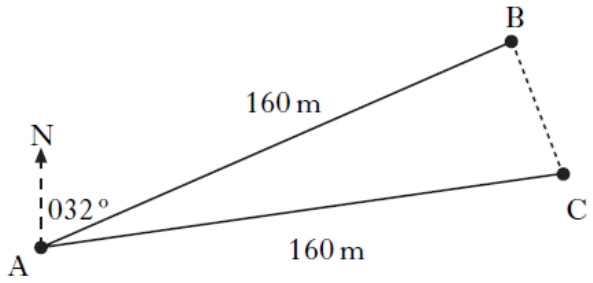
35. In triangle PQR:

- QR = 6 centimetres
- angle PQR = 30 °
- area of triangle PQR = 15 square centimetres.



Calculate the length of PQ.

36. Jane is taking part in an orienteering competition.



She should have run 160 metres from A to B on a bearing of 032°.

However, she actually ran 160 metres from A to C on a bearing of 052°.

(*a*) Write down the size of angle BAC.

(*b*) Calculate the length of BC.

(*c*) What is the bearing from C to B?

37. The weight, W kilograms, of a giraffe is related to its age, M months, by the formula

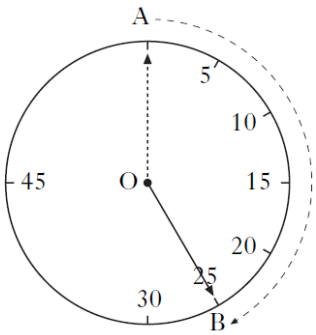
$$W = \frac{1}{4}\left(M^2 - 4M + 272\right).$$

At what age will a giraffe weigh 83 kilograms?

38. Contestants in a quiz have 25 seconds to answer a question.

This time is indicated on the clock.

The tip of the clock hand moves through the arc AB as shown.

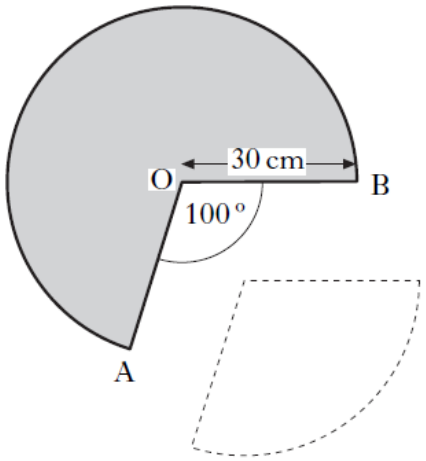


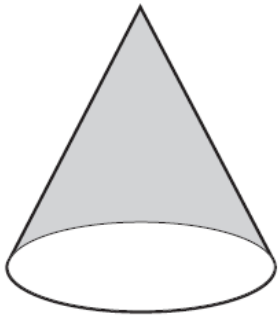
(*a*) Calculate the size of angle AOB.

(*b*) The length of arc AB is 120 centimetres.

Calculate the length of the clock hand.

39. A cone is formed from a paper circle with a sector removed as shown.
The radius of the paper circle is 30 cm.
Angle AOB is 100°.





(*a*) Calculate the area of paper used to make the cone.

(*b*) Calculate the circumference of the base of the cone.

40. The n^{th} term, T_n of the sequence 1, 3, 6, 10, . . . is given by the formula:

$$T_n = \frac{1}{2}n(n+1)$$

1^{st} term $\quad T_1 = \frac{1}{2}\times 1(1+1) = 1$

2^{nd} term $\quad T_2 = \frac{1}{2}\times 2(2+1) = 3$

3^{rd} term $\quad T_3 = \frac{1}{2}\times 3(3+1) = 6$

(*a*) Calculate the 20^{th} term, T_{20}.

(*b*) Show that $T_{n+1} = \frac{1}{2}\left(n^2+3n+2\right)$.

(*c*) Show that $T_n + T_{n+1}$ is a square number.

41. Aaron saves 50 pence and 20 pence coins in his piggy bank.

Let x be the number of 50 pence coins in his bank.

Let y be the number of 20 pence coins in his bank.

(*a*) There are 60 coins in his bank.

Write down an equation in x and y to illustrate this information.

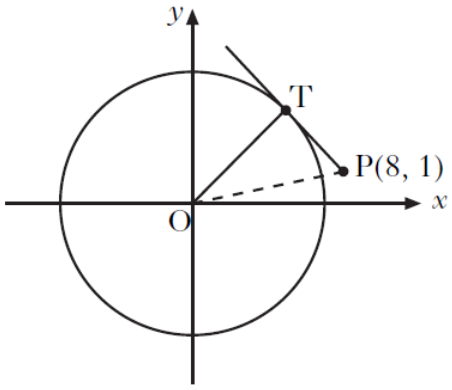
(*b*) The total value of the coins is £17·40.

Write down another equation in x and y to illustrate this information.

(*c*) Hence find **algebraically** the number of 50 pence coins Aaron has in his piggy bank.

42. A circle, centre the origin, is shown.

P is the point (8, 1).



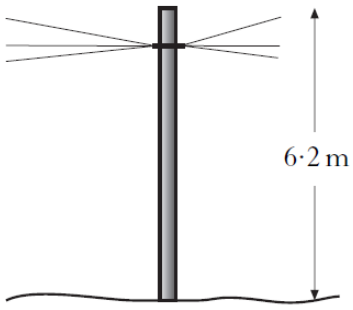
(*a*) Calculate the length of OP.

The diagram also shows a tangent from P which touches the circle at T.

The radius of the circle is 5 units.

(*b*) Calculate the length of PT.

43. A telegraph pole is 6·2 metres high.



The wind blows the pole over into the position as shown below.

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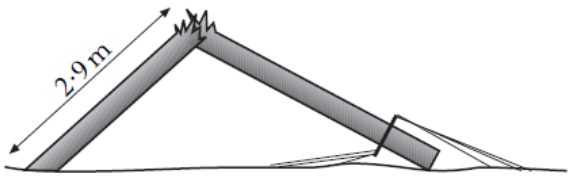
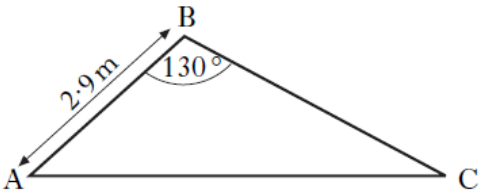
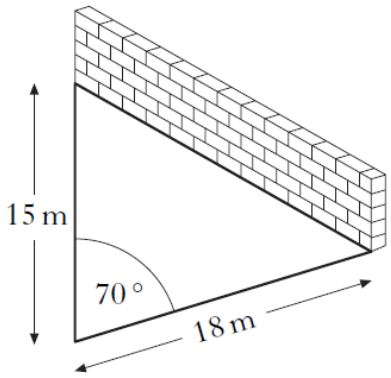




AB is 2·9 metres and angle ABC is 130°.
Calculate the length of AC.

44. A farmer builds a sheep-pen using two lengths of fencing and a wall.

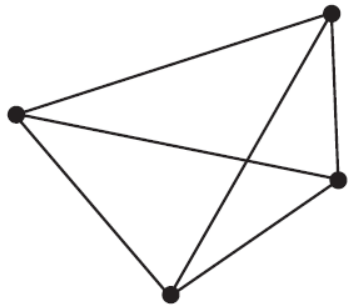


The two lengths of fencing are 15 metres and 18 metres long.

(*a*) Calculate the area of the sheep-pen, when the angle between the fencing is 70°.

(*b*) What angle between the fencing would give the farmer the largest possible area?

45. The minimum number of roads joining 4 towns to each other is 6 as shown.



The minimum number of roads, r, joining n towns to each other is given by the formula

$$r = \tfrac{1}{2}n(n-1).$$

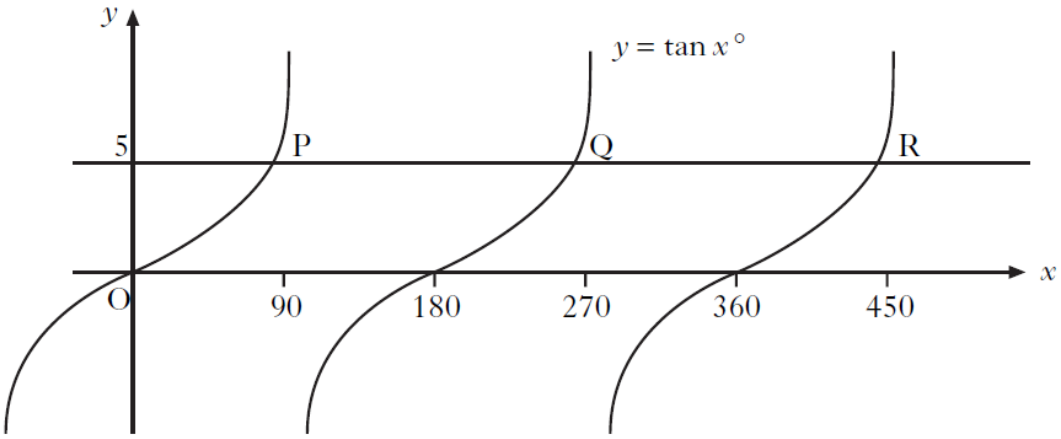
(*a*) State the minimum number of roads needed to join 7 towns to each other.

(*b*) When $r = 55$, show that $n^2 - n - 110 = 0$.

(*c*) Hence find **algebraically** the value of n.

46. The diagram shows part of the graph of $y = \tan x°$.

The line $y = 5$ is drawn and intersects the graph of $y = \tan x°$ at P and Q.



(*a*) Find the x-coordinates of P and Q.

(*b*) Write down the x-coordinate of the point R, where the line $y = 5$ next intersects the graph of $y = \tan x°$.

47. (*a*) During his lunch hour, Luke records the number of birds that visit his bird-table.

The numbers recorded last week were:

28 32 14 19 18 26 31.

Find the mean and standard deviation for this data.

(*b*) Over the same period, Luke's friend, Erin also recorded the number of birds visiting her bird-table.

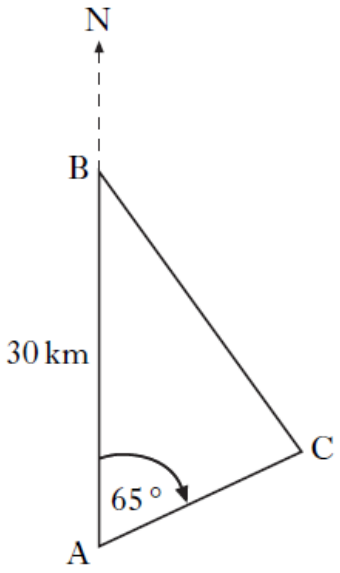
Erin's recordings have a mean of 25 and a standard deviation of 5.

Make **two** valid comparisons between the friends' recordings.

48. Brunton is 30 kilometres due North of Appleton.

From Appleton, the bearing of Carlton is 065 °.

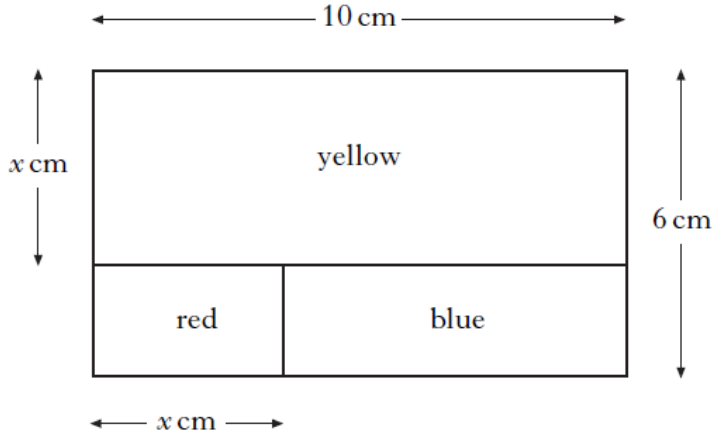
From Brunton, the bearing of Carlton is 153 °.



Calculate the distance between Brunton and Carlton.

49. (*a*) A decorator's logo is rectangular and measures 10 centimetres by 6 centimetres.

It consists of three rectangles: one red, one yellow and one blue.



The yellow rectangle measures 10 centimetres by x centimetres.

The width of the red rectangle is x centimetres.

Show that the area, A, of the blue rectangle is given by the expression

$$A = x^2 - 16x + 60.$$

(*b*) The area of the blue rectangle is equal to $\frac{1}{5}$ of the total area of the logo.

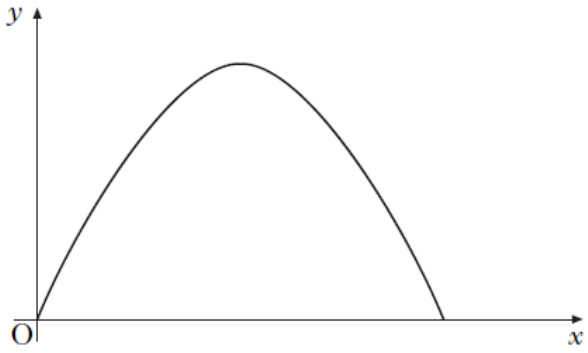
Calculate the value of x.

50. The profit made by a publishing company of a magazine is calculated by the formula

$$y = 4x(140 - x),$$

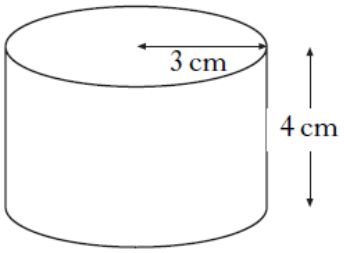
where y is the profit (in pounds) and x is the selling price (in pence) of the magazine.

The graph below represents the profit y against the selling price x.



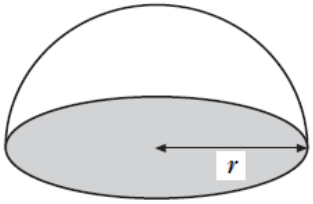
Find the maximum profit the company can make from the sale of the magazine.

51. (*a*) A cylindrical paperweight of radius 3 centimetres and height 4 centimetres is filled with sand.



Calculate the volume of sand in the paperweight.

(*b*) Another paperweight, in the shape of a hemisphere, is filled with sand.



It contains the same volume of sand as the first paperweight.

Calculate the radius of the hemisphere.

[END OF SECTION D]

Section D - Answers

1. $\bullet^1$ $72\% = 1296$

$\bullet^2$ $1\% = \dfrac{1296}{72} = 18$

$\bullet^3$ $18 \times 28 = 504$

2. $\bullet^1$ $9 = \dfrac{1}{2} \times x^2 \times \sin 30^\circ$

$\bullet^2$ 36

$\bullet^3$ $x = 6$

3. $\bullet^1$ $0{\cdot}93$

$\bullet^2$ $0{\cdot}93 \times 94 = 87{\cdot}42$

$\bullet^3$ … $75{\cdot}6$

$\bullet^4$ During April or 4^{th} month

4. $\bullet^1$ $\dfrac{x}{360}$

$\bullet^2$ $\dfrac{36{\cdot}7}{100\pi}$

$\bullet^3$ $42°$

5. $\bullet^1$ 4×2^3

$\bullet^2$ 32

$\bullet^1$ $4 = 4 \times 2^m$

$\bullet^2$ $m = 0$

6. $\bullet^1$ $(2x)^2 = x^2 + 6^2 - 2 \times x \times 6 \times 0{\cdot}5$

$\bullet^2$ $4x^2 = x^2 - 6x + 36$

$\bullet^3$ $x^2 + 2x - 12 = 0$

7. $\bullet^1$ use of Pythagoras

$\bullet^2$ $30^2 - 27{\cdot}5^2$

$\bullet^3$ $11{\cdot}99$

$\bullet^4$ $18{\cdot}01$ $\bullet^1$ $41{\cdot}99$

8. $\bullet^1$ $1 + \sin x^\circ = 1{\cdot}7$

$\bullet^2$ $\sin x^\circ = 0{\cdot}7$

$\bullet^3$ $44{\cdot}4^\circ$

$\bullet^4$ $135{\cdot}6^\circ$

9. • $\dfrac{1}{2}\, ab \sin C$

• $\dfrac{1}{2} \times 40 \times 40 \times \sin 110°$

• $751{\cdot}75$

• $1503{\cdot}5$

10. $\bullet^1$ $\dfrac{25}{40}$ or $\dfrac{35}{45+x}$

$\bullet^2$ $\dfrac{25}{40} = \dfrac{35}{45+x}$

$\bullet^3$ $25(45 + x) = 35 \times 40$

$\bullet^4$ 11

11. • 14

• on average the **number** of sit-ups per athlete has risen

• the number of sit-ups is less varied

12. • $\angle BAC = 121°$

• $\dfrac{a}{\sin 121°} = \dfrac{8}{\sin 22°}$

• $a = \dfrac{8 \sin 121°}{\sin 22°}$

• $a = 18{\cdot}3$

13. • $1{\cdot}2$

• $1{\cdot}8$

• $f = 1{\cdot}2d + 1{\cdot}8$

• $1{\cdot}2 \times 7 + 1{\cdot}8$

• $10{\cdot}2(0)$

14.
- $AC^2 = 6 \cdot 2^2 + 4^2$
- $7 \cdot 38$
- cosine rule
- $\cos D = \dfrac{5^2 + 7^2 - 54 \cdot 44}{2 \times 5 \times 7}$
- $\cos D = 0 \cdot 279\cdots$
- $73 \cdot 8°$

15.
- -3
- $3 \sin t° = 0 \cdot 6$
- $\sin t° = 0 \cdot 2$
- $11 \cdot 5°$
- $168 \cdot 5°$

16.
- 5:6

17.
- $\dfrac{2x}{x^2 + 5} = \dfrac{6}{18}$ or $3 \times 2x = x^2 + 5$
- $x^2 - 6x + 5 = 0$
- $(x - 1)(x - 5) = 0$
- $x = 1, \quad x = 5$
- $x = 5$

18.
- $\dfrac{42}{360}$
- $\dfrac{42}{360} \times \pi \times 2{\cdot}4$
- 0·879
- no, as $0{\cdot}879 < 0{\cdot}9$

19.
- $\frac{125}{90}$
- $\left(\frac{125}{90}\right)^2$
- 7754·6
- no, as 7754·6 ≠ 8040
  (8040 must be explicit)

20.
- recognition of right angle at chord
- $r^2 = (r - 5)^2 + 9^2$
- 10·6

21.
- 1200
- $V = 1200 \times l$
- $156\,000 = 1200 \times l$
- 130

22.
- $l = \dfrac{3000}{70} (= 42{\cdot}86)$
- $b = \dfrac{42{\cdot}86}{3} (= 14{\cdot}29)$
- $b = \pi d$
- d = 4·55

23.
- $(90, 1)$
- $4 \sin x° - 3 = 0$
- $\sin x° = \dfrac{3}{4}$
- 48·6°

24.
- $48 + 8t - t^2 = 0$
- $(4 + t)(12 - t) = 0$
- -4 , 12
- 12

25.
- 1·69
- the second sample has on average, a greater number of pins per box
- the second sample has a greater variability in the number of pins per box

26.
- $1^2 = x^2 + 0 \cdot 5^2$
- $x = 0 \cdot 866\cdots$
  - $0 \cdot 5 \times 0 \cdot 5 \times 0 \cdot 866 + 0 \cdot 5 \times (2 - 0 \cdot 866)$
  - $0 \cdot 7835 \times 2$
  - 1·567

27.
- 64°
- $\frac{e}{\sin 64^\circ} = \frac{26.2}{\sin 34^\circ}$ or

$e^2 = 26 \cdot 2^2 + 46 \cdot 4^2 - 2 \times 26 \cdot 2 \times 46 \cdot 4 \cos 64^\circ$
- $42 \cdot 1$
- 9

28.
- $\frac{1}{2} \times 6 \times 5 \times \sin x^\circ = 12$
- $\sin x^\circ = \frac{12}{15}$
- $x = \sin^{-1}\left(\frac{12}{15}\right) = 53 \cdot 1^\circ$
- $126 \cdot 9^\circ$

29.
- $(x+8)^2 = x^2 + (x+7)^2$
- $x^2 + 16x + 64 = 2x^2 + 14x + 49$
- $x^2 - 2x - 15 = 0$
- $(x-5)(x+3)$
- $x = 5$

30.
- $3 + 1 \cdot 75 \sin(30 \times 5)^\circ$
- $3 \cdot 875$
- $1 \cdot 25$ or $4 \cdot 75$
- $3 \cdot 5$

31.
- knows to use $\pi r^2 h$
- $\pi \times 3^2 \times 8 \ (= 226.19)$
- $\frac{2000}{72\pi} \ (= 8.84)$
- 8 (rounding down)

32.
- 372
- 74

33.
- $y = 2x + 3$
- $2 \times 20 + 3$
- 43

34.
- 0.8
- 0.8^3
- 0.512
- no, because $51.2\% > 50\%$

35.
- $\mathrm{A} = \frac{1}{2} pr \sin Q$
- $15 = \frac{1}{2} \times 6 \times r \times \sin 30^\circ$
- 10

36.
- 20°
- 55.6
- 312°

37.
- $83 = \cdots$
- $M^2 - 4M - 60 = 0$
- $(M-10)(M+6) = 0$
- 10

38.
- $\frac{260}{360}$
- $\frac{260}{360}\times\pi\times30^2$
- 2042
- circumference of base = length of arc
- $C=\frac{260}{360}\times\pi\times60$
- 136.1

39.
- 210
- $\frac{1}{2}(n+1)(n+2)$
- $\frac{1}{2}(n^2+3n+2)$

40.
- $x+y=60$
- $50x+20y=1740$
- $20x+20y=1200$ or equivalent
- $30x=540$ or equivalent
- 18

41.
- $OP^2=8^2+1^2$
- $\sqrt{65}$ or 8.06
- $PT^2=\left(\sqrt{65}\right)^2-5^2$
- $\sqrt{40}$ or 6.32

42.
- BC = 3.3
- use of cosine rule
- $AC^2=2.9^2+3.3^2-2\times2.9\times3.3\cos130^\circ$
- 5.62

43.
- $\frac{1}{2}ab\sin C$
- $\frac{1}{2}\times15\times18\times\sin\ 70^\circ$
- 126.9
- 90°

44.
- 150°

	or
$\frac{150}{360}$	$\frac{5}{12}$
$\frac{150}{360}=\frac{120}{2\pi r}$	$\frac{120}{5/12}=288$
$r=\frac{360}{150}\times\frac{120}{2\pi}$	$r=\frac{288}{2\pi}$
45.8	45.8

45.
- 21
- $55=\frac{1}{2}n(n-1)$
- $n^2-n-110=0$
- $(n+10)(n-11)=0$
- -10 and 11
- 11

46.
- $\tan x^\circ=5$
- 78.7
- 258.7 (first solution + 180)
- 438.7

47.
- 24
- 7
- on average, more birds visit Erin's table
- the number of birds visiting Luke's table varies more

48.

- $\angle ABC = 27°$
- third angle and use of sine rule
- $\dfrac{a}{\sin 65^\circ} = \dfrac{30}{\sin 88^\circ}$
- 27·2

49.

- $A = (10 - x)(6 - x)$

  or

  $A = 60 - 10x - x(6 - x)$
- $A = x^2 - 16x + 60$
- $x^2 - 16x + 60 = 12$
- $(x - 4)(x - 12)$
- 4, 12
- 4

50.

- $V = \pi \times 3^2 \times 4$
- 113·1
- $\dfrac{2}{3}\pi \times r^3 = 113 \cdot 1$
- $r^3 = 54$
- 3·78

51.

- $4x(140 - x) = 0$
- 0, 140
- 70
- 19 600

Section E

Further questions by syllabus area (* no calculator)

A1

1.* Multiply out the brackets and simplify:

(a) $(x-2)(x^2+4x+1)$ (b) $2x(x-3)^2$ (c) $(3x+2)^2-9(x^2-1)$

A2

1.* Factorise fully:

(a) $6pqr-15qs$ (b) t^2-36 (c) $p^2-2p-15$

(d) $3x^2-13x-10$ (e) $4x^2-20x+9$ (f) $3k^2-75$

(g) x^2-9x^4 (h) $4k^2-12k-40$ (i) x^3+7x^2-12x

A3

1.* Express the following in the form $(x+a)^2+b$:

(a) x^2+4x+9 (b) x^2+6x+5 (c) x^2+2x-5

(d) $x^2-8x+19$ (e) $x^2-10x-2$ (f) x^2+x+1

A4

1.* Reduce each of the following algebraic fractions to its simplest form:

(a) $\dfrac{x^2-4}{x+2}$ (b) $\dfrac{2x+6}{x^2+3x}$ (c) $\dfrac{t^2-5t}{t^2-3t-10}$

(d) $\dfrac{p^2+4p+3}{p^2-9}$ (e) $\dfrac{2x^2+4x^3}{1-4x^2}$ (f) $\dfrac{x^2-3x-18}{x^2+x-42}$

A5

1.* Carry out the following calculations, expressing your answer as a single fraction in its simplest form:

(a) $\dfrac{x}{5}+\dfrac{3x}{5}$ (b) $\dfrac{x}{3}+\dfrac{x}{4}$ (c) $\dfrac{p}{3}-\dfrac{p}{7}$

(d) $\dfrac{3t}{4}-\dfrac{t}{2}$ (e) $\dfrac{x+1}{3}+\dfrac{5x-2}{2}$ (f) $\dfrac{2x}{3}-\dfrac{x-1}{5}$

(g) $\dfrac{5}{x}-\dfrac{4}{x^2}$ (h) $\dfrac{3}{x-2}+\dfrac{4}{x+6}$ (i) $\dfrac{x}{x+5}-\dfrac{x-1}{x-4}$

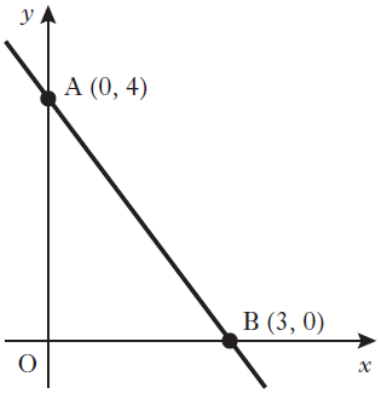
2.* Carry out the following calculations, expressing your answer as a single fraction in its simplest form:

(a) $\frac{3x}{y^3} \times \frac{y}{x^2}$ (b) $\frac{2p^3q}{r} \times \frac{6r^2}{4pq}$ (c) $\frac{x-1}{5} \times \frac{4}{(x-1)^2}$

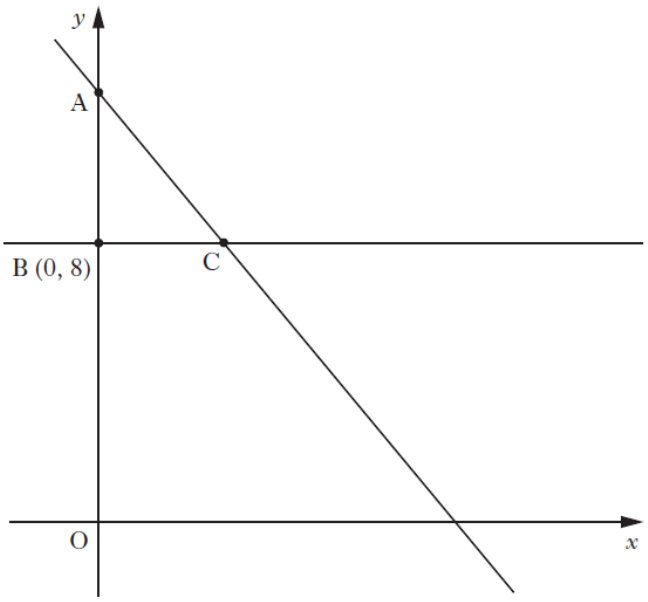
(d) $\frac{3}{x} \div \frac{6}{x^2}$ (e) $\frac{3pq}{5} \div \frac{6q}{p}$ (f) $4 \div \frac{t^2}{5}$

A6

1.* Find the equation of the straight line AB.



2.* The straight line with equation $4x + 3y = 36$ cuts the y-axis at A.

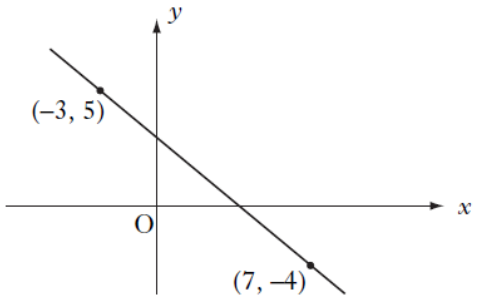


(*a*) Find the coordinates of A.

This line meets the line through B (0, 8), parallel to the x-axis, at C as shown above.

(*b*) Find the coordinates of C.

3.*



Calculate the gradient of the straight line passing through the points (–3, 5) and (7, –4).

A7

1.* Solve the following equations/inequations:

(a) $20-2(3x+8)=8-5x$

(b) $2-(6-4x)\leq 15-9(2x+7)$

(c) $2(1-5p)-12=10-6p$

(d) $5(2-x)-3(2-3x)\geq 2+3x$

(e) $13-4x<18-7(2-x)$

(f) $15-(3-6x)<9x+10$

(g) $\frac{1}{5}(4x+3)+\frac{1}{3}(2x+3)=6$

(h) $\frac{x+5}{3}=3+\frac{3x-2}{4}$

A8

1.* Solve algebraically the system of equations

$$2x - y = 10$$
$$4x + 5y = 6.$$

2. Solve algebraically the system of equations

$$2x - 5y = 24$$
$$7x + 8y = 33.$$

3. Three groups are booking a holiday. The first group consists of 6 adults and 2 children. The total cost of their holiday is £3148.

Let x pounds be the cost for an adult and y pounds be the cost for a child.

(*a*) Write down an equation in x and y which satisfies the above information.

The second group books the same holiday for 5 adults and 3 children. The total cost of their holiday is £3022.

(*b*) Write down a second equation in x and y which satisfies this information.

(*c*) The third group books the same holiday for 2 adults and 4 children. The travel agent calculates that the total cost is £2056.

Has this group been overcharged?

Justify your answer.

4. Alan is taking part in a quiz. He is awarded x points for each correct answer and y points for each wrong answer. During the quiz, Alan gets 24 questions correct and 6 wrong. He scores 60 points.

(a) Write down an equation in x and y which satisfies the above condition.

Helen also takes part in the quiz. She gets 20 questions correct and 10 wrong. She scores 40 points.

(b) Write down a second equation in x and y which satisfies this condition.

(c) Calculate the score for David who gets 17 correct and 13 wrong.

A9

1.* Change the subject of the formula

$$a = 3b^2 + c$$

to b.

2.* A formula used to calculate lighting efficiency is

$$E = \frac{I}{D^2}.$$

Change the subject of this formula to D.

3.* Change the subject of the formula

$$P = 2(L + B)$$

to L.

4.* Change the subject of the formula

$$A = \frac{1}{2}h(a+b)$$

to h.

5.* Change the subject of the formula

$$p = q + \sqrt{a}$$

to a.

6.* Change the subject of the formula

$$K = \frac{m^2 n}{p}$$

to m.

A10 – A13

1. Solve the equation

$$x^2 - 5x - 2 = 0,$$

giving the roots correct to one decimal place.

2. Solve the equation

$$3x^2 + 7x - 5 = 0,$$

giving the roots correct to one decimal place.

3. Solve the equation

$$x^2 + 5x + 3 = 0,$$

giving the roots correct to one decimal place.

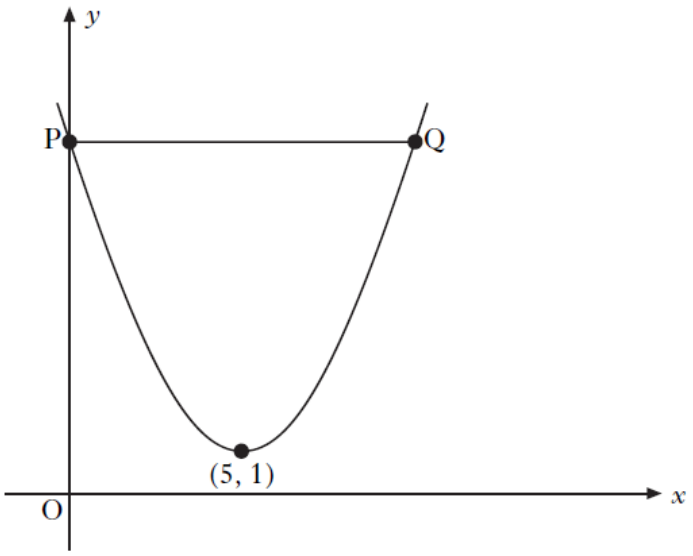
4 Solve the equation

$$5x^2 + 4x - 2 = 0,$$

giving the roots correct to 2 decimal places.

5.* The graph below shows part of a parabola with equation of the form

$$y = (x + a)^2 + b.$$

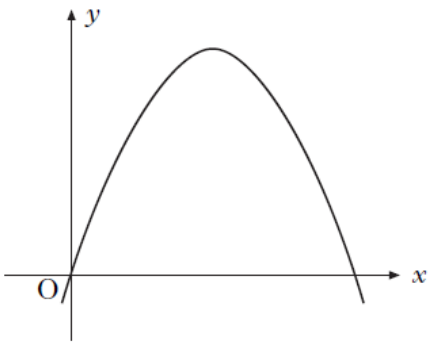


(*a*) State the values of a and b.

(*b*) State the equation of the axis of symmetry of the parabola.

(*c*) The line PQ is parallel to the x-axis.
Find the coordinates of points P and Q.

6.* The graph shown below is part of the parabola with equation $y = 8x - x^2$.



(*a*) By factorising $8x - x^2$, find the roots of the equation

$$8x - x^2 = 0.$$

(*b*) State the equation of the axis of symmetry of the parabola.

(*c*) Find the coordinates of the turning point.

7.* The diagram below represents a rectangular garden with length $(x + 7)$ metres and breadth $(x + 3)$ metres.

(*a*) Show that the area, A square metres, of the garden is given by

$$A = x^2 + 10x + 21.$$

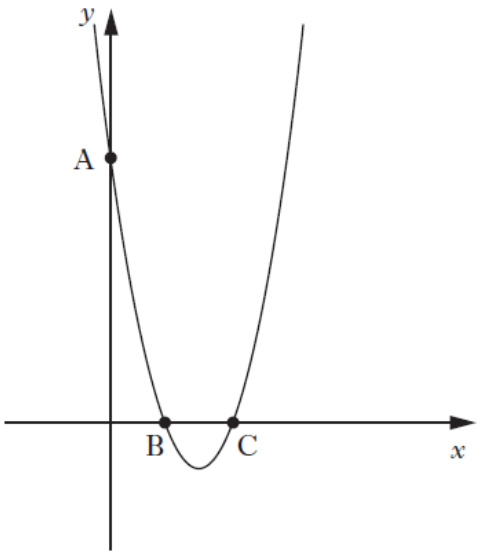
(*b*) The area of the garden is 45 square metres. Find x.

Show clearly all your working.

8.* The equation $x^2 - 6x + 8 = 0$ can also be written as $(x - 2)(x - 4) = 0$.

(*a*) Write down the roots of the equation $x^2 - 6x + 8 = 0$.

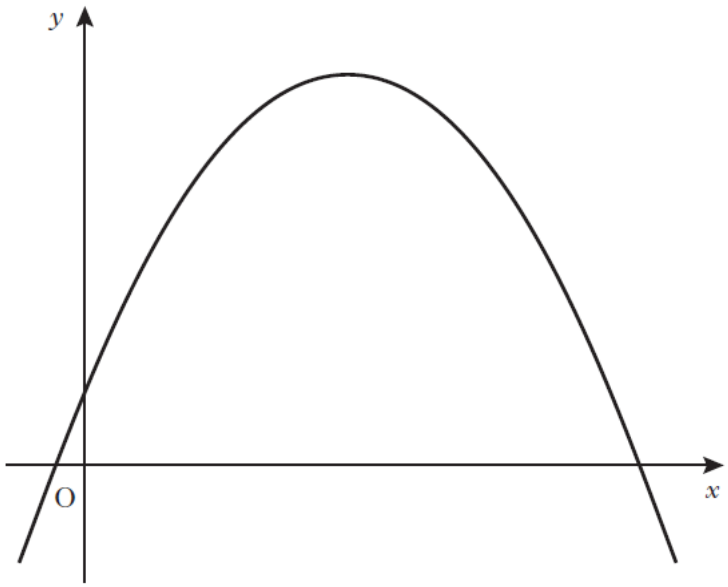
Part of the graph of $y = x^2 - 6x + 8$ is shown below.



(*b*) State the coordinates of the points A, B and C.

(*c*) What is the equation of the axis of symmetry of this graph?

9.* The diagram below shows part of the graph of $y = 20 - (x - 4)^2$.



(*a*) State the coordinates of the maximum turning point.

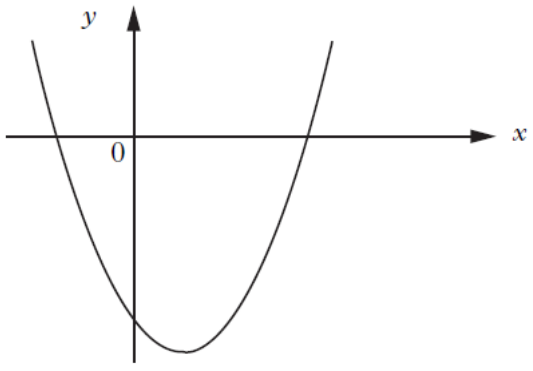
(*b*) State the equation of the axis of symmetry.

10.* (*a*) Factorise $x^2 - 4x - 21$.

(*b*) Hence write down the roots of the equation

$$x^2 - 4x - 21 = 0.$$

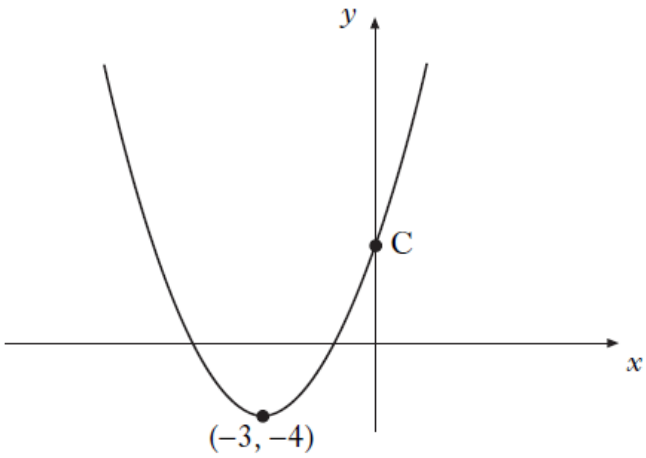
(*c*) The graph of $y = x^2 - 4x - 21$ is shown in the diagram.



Find the coordinates of the turning point.

11.* The diagram below shows part of a parabola with equation of the form

$$y = (x + a)^2 + b.$$

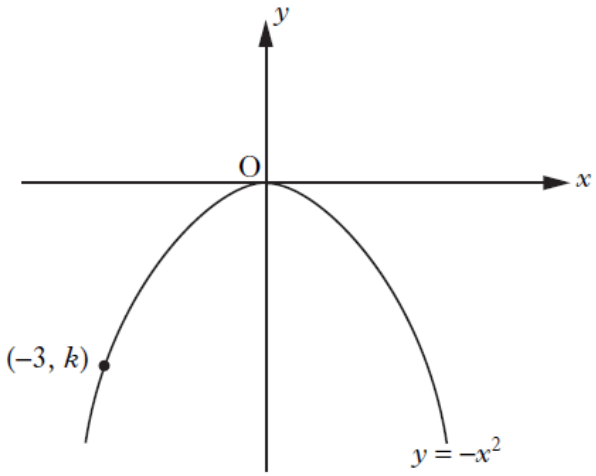


(*a*) Write down the equation of the axis of symmetry of the graph.

(*b*) Write down the equation of the parabola.

(*c*) Find the coordinates of C.

12.* The diagram below shows the graph of $y = -x^2$.



The point $(-3, k)$ lies on the graph.

Find the value of k.

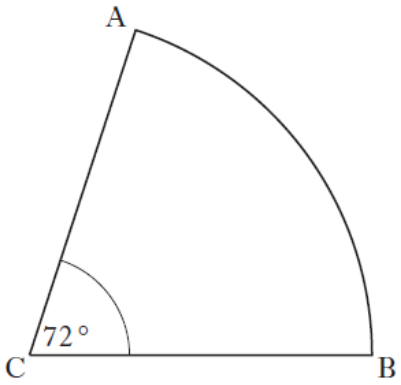
13.* Find the values of k such that the equation $2x^2 - kx + 2 = 0$ has equal roots.

14.* Show that the equation $x^2 - 5x + 7 = 0$ has no real roots.

15.* A parabola has equation $y = tx^2 + 6x - t$. Show that for all values of t the parabola intersects the x-axis at two distinct points.

G2

1.* The diagram below shows a sector of a circle, centre C.

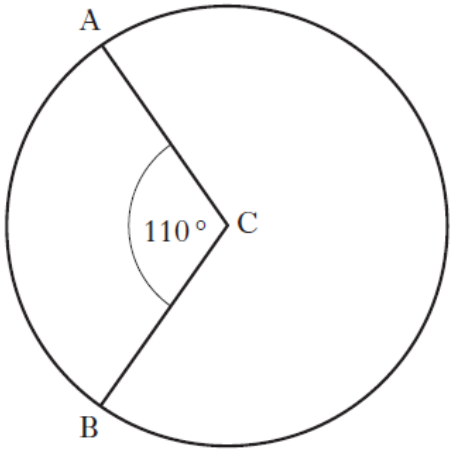


The radius of the circle is 5 centimetres and angle ACB is 72°.

Calculate the length of arc AB.

Take $\pi = 3{\cdot}14$.

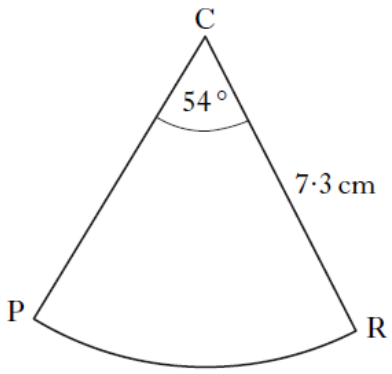
2. The diagram below shows a circle, centre C.



The circumference of the circle is 40·8 centimetres.

Calculate the length of the minor arc AB.

3. The diagram below shows a sector of a circle, centre C.



The radius of the circle is 7·3 centimetres and angle PCR is 54°.

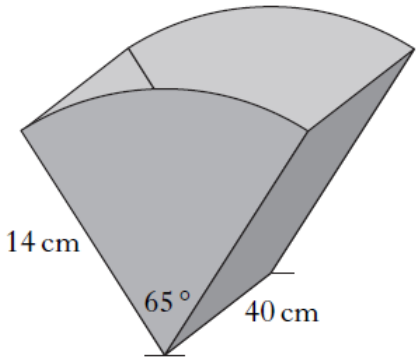
Calculate the area of the sector PCR.

4. The ends of a magazine rack are identical.

Each end is a sector of a circle with radius 14 centimetres.
The angle in each sector is 65°.

The sectors are joined by two rectangles, each with length 40 centimetres.

The exterior is covered by material.
What area of material is required?



G3

1. The Battle of Largs in 1263 is commemorated by a monument known as The Pencil.

This monument is in the shape of a cylinder with a cone on top.

The cylinder part has diameter 3 metres and height 15 metres.

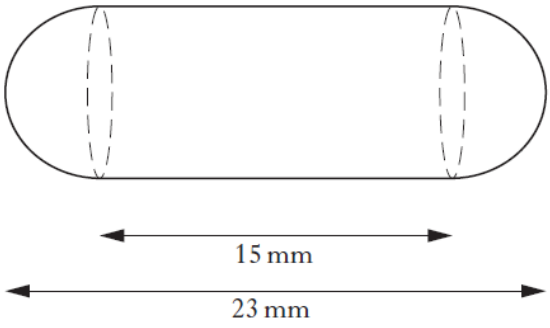
(*a*) Calculate the volume of the **cylinder** part of The Pencil.

The volume of the **cone** part of The Pencil is 5·7 cubic metres.

(*b*) Calculate the **total** height of The Pencil.

2. A health food shop produces cod liver oil capsules for its customers.

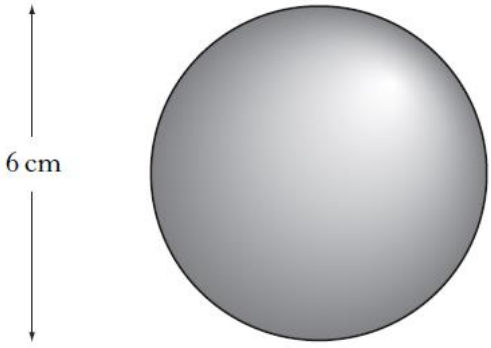
Each capsule is in the shape of a cylinder with hemispherical ends as shown in the diagram below.



The total length of the capsule is 23 millimetres and the length of the cylinder is 15 millimetres.

Calculate the volume of one cod liver oil capsule.

3.* The diagram below represents a sphere.

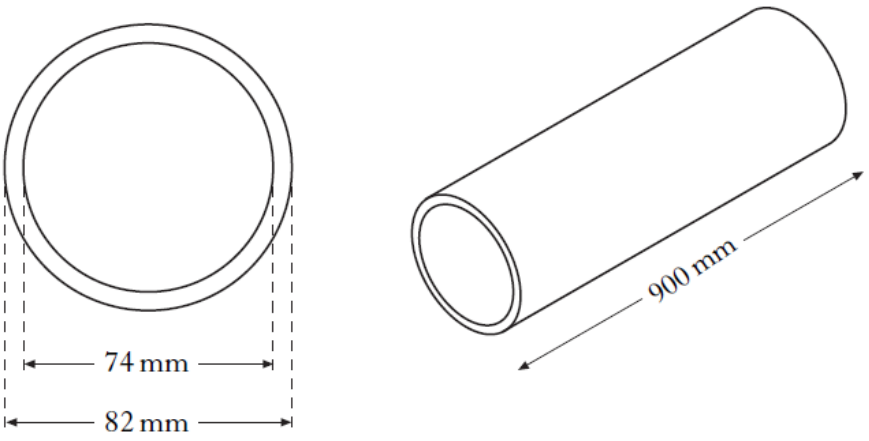


The sphere has a diameter of 6 centimetres.

Calculate its volume.

Take $\pi = 3{\cdot}14$.

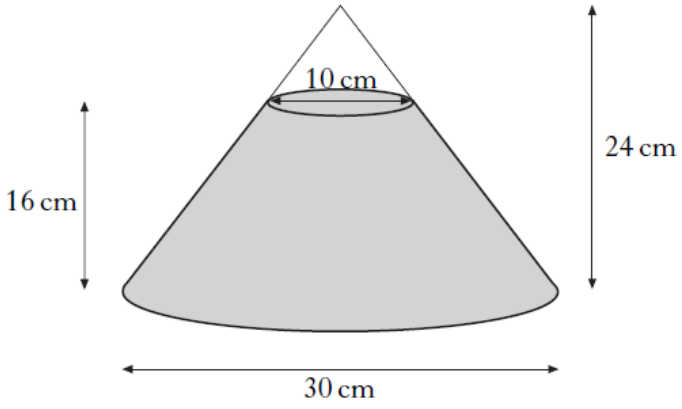
4. A company manufactures aluminium tubes.
The cross-section of one of the tubes is shown in the diagram below.



The inner diameter is 74 millimetres.
The outer diameter is 82 millimetres.
The tube is 900 millimetres long.

Calculate the volume of aluminium used to make the tube.
Give your answer correct to three significant figures.

5. A glass ornament in the shape of a cone is partly filled with coloured water.



The cone is 24 centimetres high and has a base of diameter 30 centimetres.
The water is 16 centimetres deep and measures 10 centimetres across the top.
What is the volume of the water?
Give your answer correct to 2 significant figures.

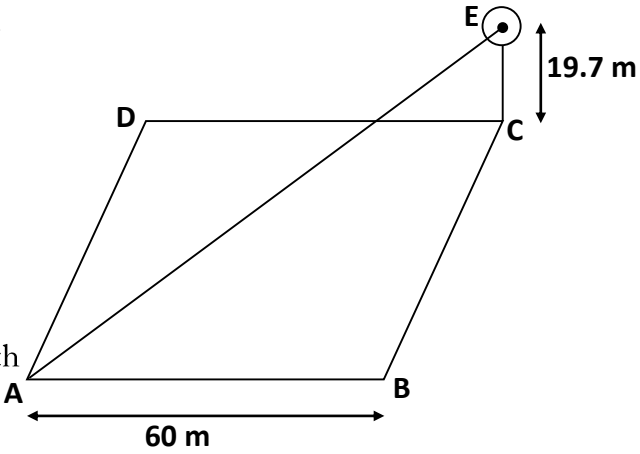
G4, G5

1. A university quadrangle is in the shape of a square ABCD with a clock tower in the corner at C.

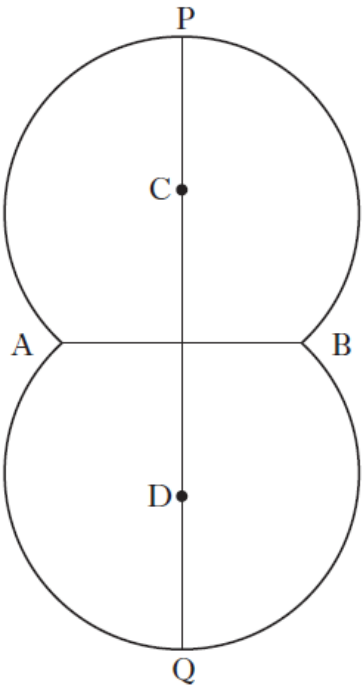
   The centre of the clock, E, is 19.7 metres above C.

   To support the tower a cable runs from the point A to the centre of the clock, E.

   Given that AB is 60 metres, calculate the length of the cable to 3 significant figures.



2. The shape below is used as a logo in an advertising campaign. It is made up from segments of two identical circles.



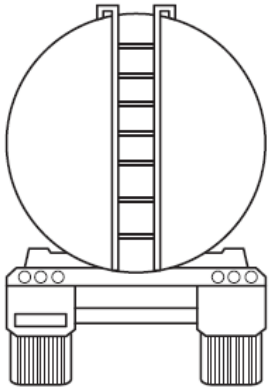
The points C and D are the centres of the circles and each circle has a radius of 24 centimetres.

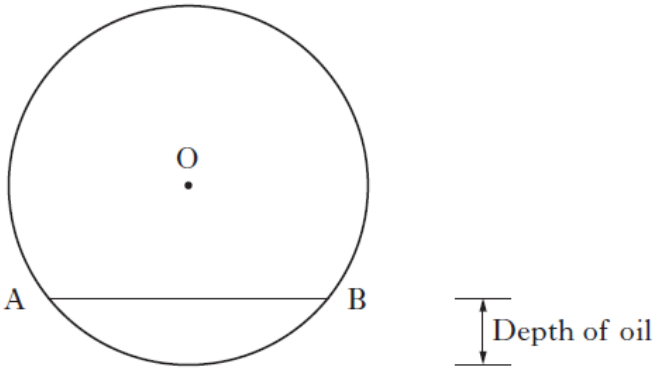
AB is a common chord of length 30 centimetres.

Calculate the height of the logo, represented by the line PQ.

3. A tanker delivers oil to garages.

The tank has a circular cross-section as shown in the diagram below.



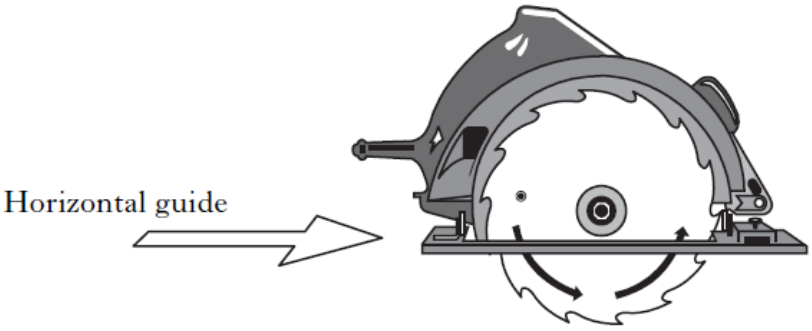


The radius of the circle, centre O, is 1·9 metres.

The width of the surface of the oil, represented by AB in the diagram, is 2·2 metres.

Calculate the depth of the oil in the tanker.

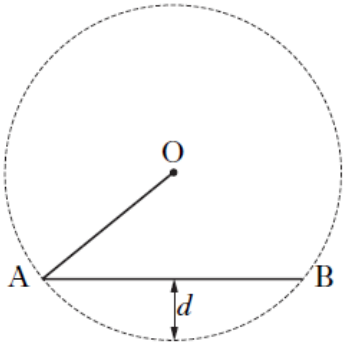
4. A circular saw can be adjusted to change the depth of blade that is exposed below the horizontal guide.



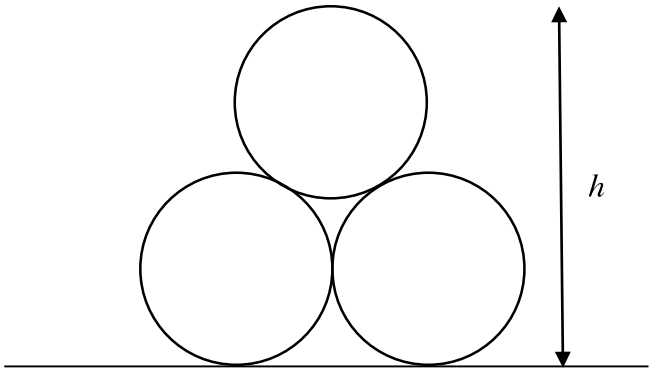
The circle, centre O, below represents the blade and the line AB represents part of the horizontal guide.

This blade has a radius of 110 millimetres.

If AB has length 140 millimetres, calculate the depth, d millimetres, of saw exposed.

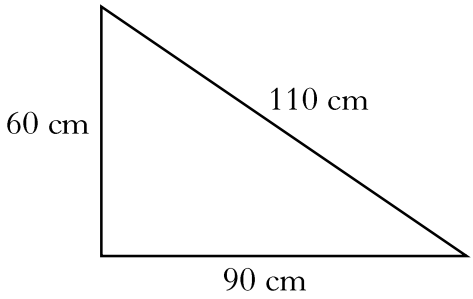


5. Three pipes are stored on horizontal ground as shown in the diagram.



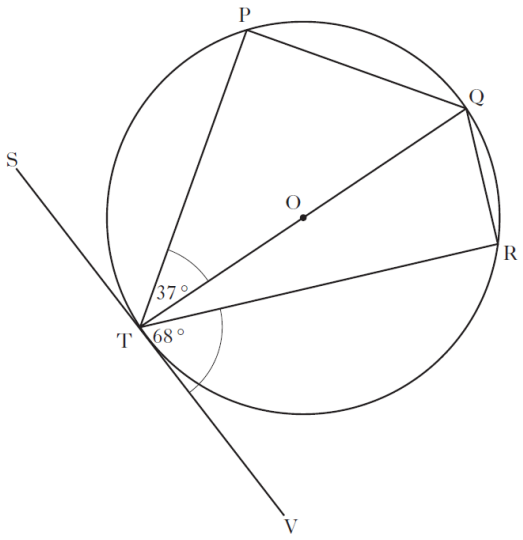
Each pipe has a circular cross-section of radius 1 metre.
Calculate the height, h, of the stacked pipes, giving your answer to **2 significant figures.**
[You may ignore the thickness of the pipes]

6. A triangular paving slab has dimensions as shown.

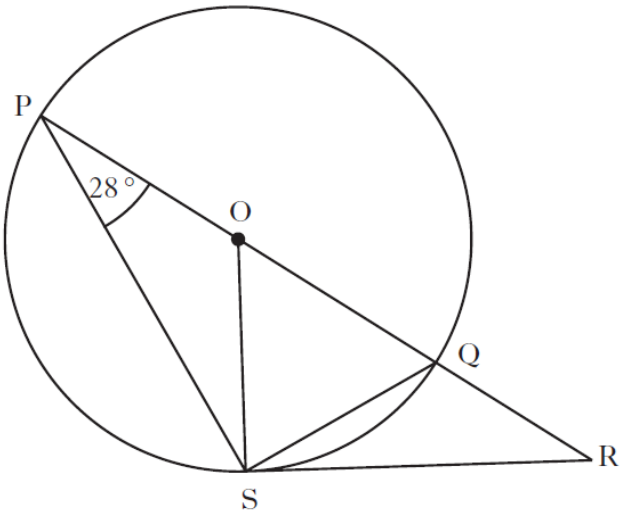


Is the paving slab in the shape of a right-angled triangle? **Explain your answer**.

7.* The tangent SV touches the circle, centre O, at T.
Angle PTQ is 37 ° and angle VTR is 68 °.
Calculate the size of angle PQR.



8.*

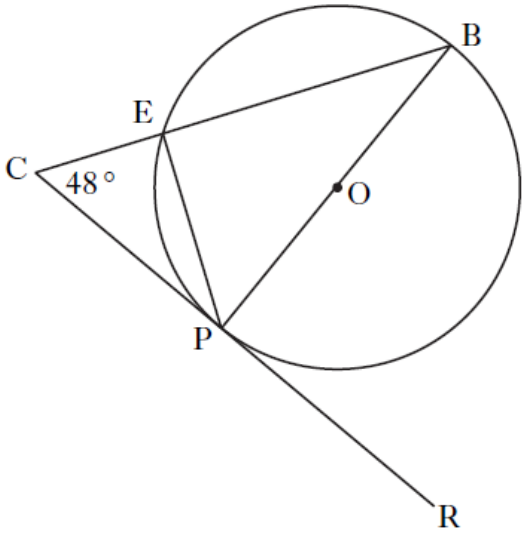


In the above diagram,

- O is the centre of the circle
- PQ is a diameter of the circle
- PQR is a straight line
- RS is a tangent to the circle at S
- angle OPS is 28°.

Calculate the size of angle QRS.

9.* A circle, centre O, is shown below.



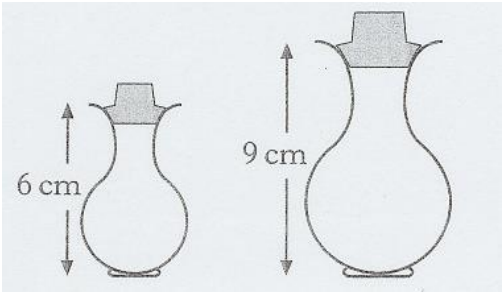
In the circle

- PB is a diameter
- CR is a tangent to the circle at point P
- Angle BCP is 48°.

Calculate the size of angle EPR.

G6

1. Two perfume bottles are mathematically similar in shape.



The smaller one is 6 cm high and holds 30 millilitres of perfume.
The larger bottle is 9 cm high.
What volume of perfume will the larger bottle hold?

2. Two oil tankers are mathematically similar.
The larger tanker is 10.5 metres long and can hold 67.5 m^3.
The smaller tanker holds 20 m^3.
How long is the smaller tanker?

3. Two plastic containers are mathematically similar.
The smaller container has a volume of 0.8 m^3 and a surface area of 1.4 m^2.
The larger container has a volume of 6.4 m^3.
What is the surface area of the larger container?

G7 – G9

1.* $\mathbf{p} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix}$.

Calculate:

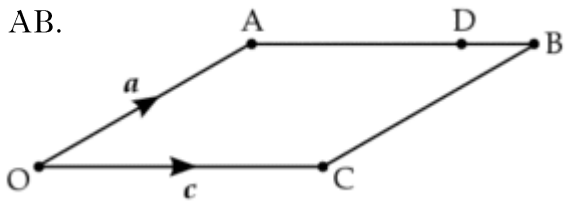
(a) $3\mathbf{p} - 2\mathbf{q}$
(b) $|\mathbf{p}|$
(c) $|\mathbf{p} + \mathbf{q}|$

2.* OABC is a parallelogram.

The sides OA and OC are represented by vectors **a** and **b** respectively.

D lies three-quarters of the way along AB.

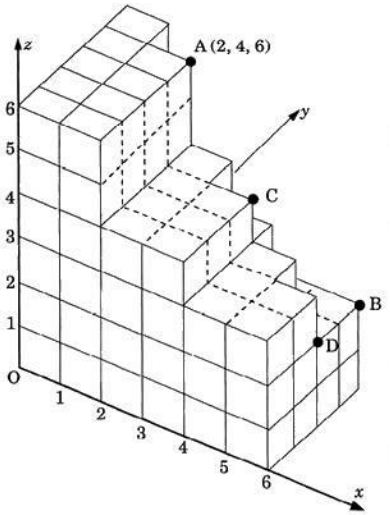
Express OD in terms of **a** and **b.**



3.* Relative to coordinates axes, the point A has coordinates (2, 4, 6).

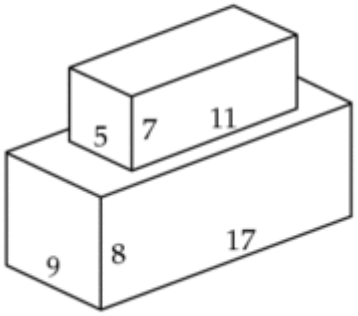
(a) Find the coordinates of C and D.

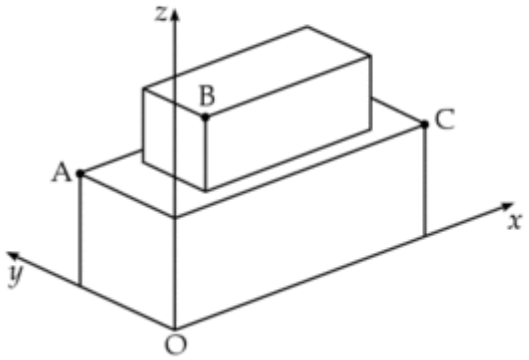
(b) Write down the coordinates of B.



4.* A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinates axes are taken as shown.

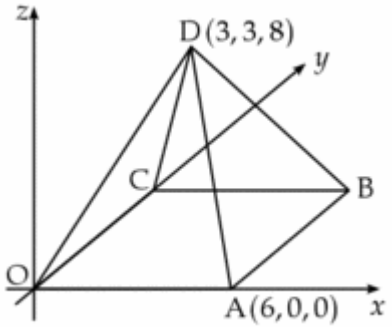




The point A has coordinates (0,9,8) and C has coordinates (17,0,8).
Write down the coordinates of B.

5.* The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units. The coordinates of A and D are (6, 0, 0) and (3, 3, 8)



(a) Write down the coordinates of B

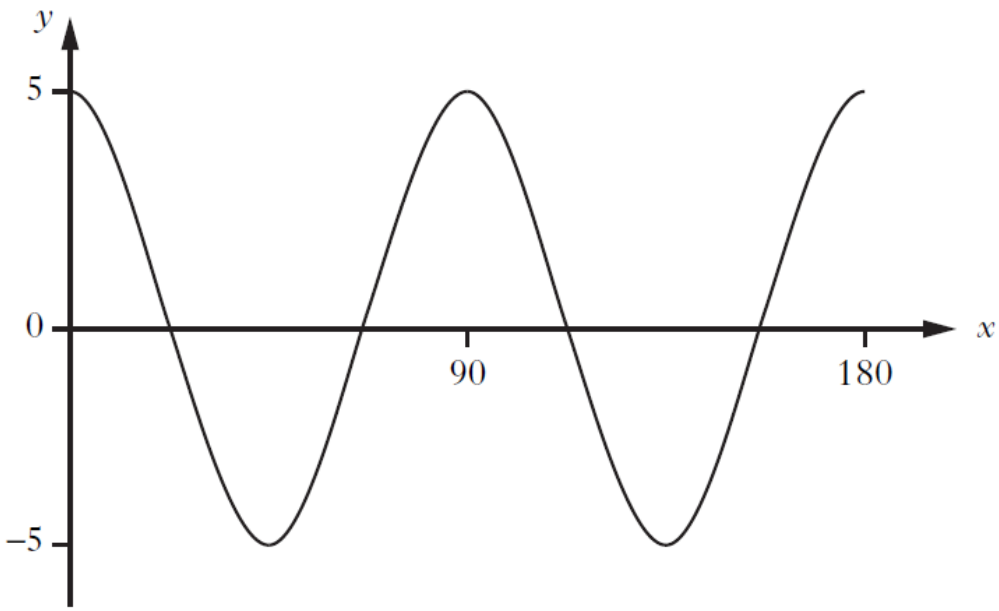
(b) M is the midpoint of AD. Write down the coordinates of M.

T1

1.* Sketch the graph of $y = \sin(x - 90)°$, $0 \leq x \leq 360$.

2.* Sketch the graph of $y = -2\sin x°$, $0 \leq x \leq 360$.

3.* Part of the graph of $y = a\cos bx°$ is shown in the diagram.



State the values of a and b.

T2

1.* State the period of $y = \sin 2x°$

2.* Simplify $\dfrac{\cos x° \tan x°}{\sin x°}$.

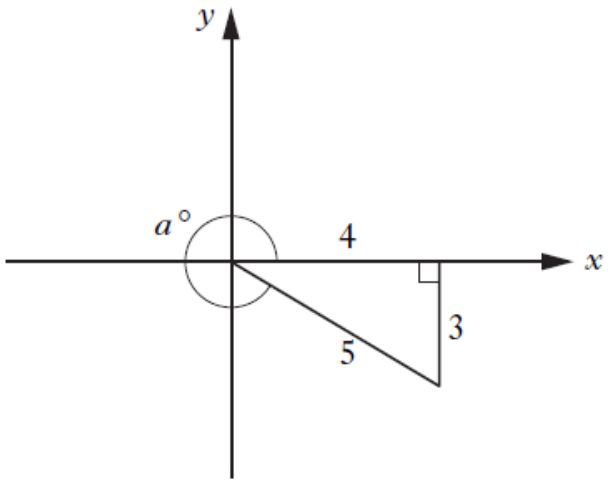
3. A Ferris wheel is turning at a steady rate.

The height, h metres, of one of the cars above the ground at a time t seconds is given by the formula

$$h = 7 + 5\sin t°.$$

Find **two** times during the first turn when the car is at a height of 10·8 metres above the ground.

4.*



Write down the value of $\cos a°$.

5. Prove that

$$\frac{\sin^2 A}{1-\sin^2 A} = \tan^2 A.$$

6. The height, h metres, of the lift above the ground is given by the formula

$$h = 15\tan x° + 1{\cdot}7,$$

where $x°$ is the angle of elevation of the lift from the surveyor at point P.

(*a*) What is the height of the lift above the ground when the angle of elevation from P is 25 °?

(*b*) What is the angle of elevation at point P when the height of the lift above the ground is 18·4 metres?

7. Simplify

$$\frac{\cos^3 x°}{1-\sin^2 x°}.$$

8. Solve the equation

$$7\sin x° + 1 = -5, \qquad 0 \leq x \leq 360.$$

9. Solve the equation

$$4\cos x° + 3 = 0, \qquad 0 \leq x \leq 360.$$

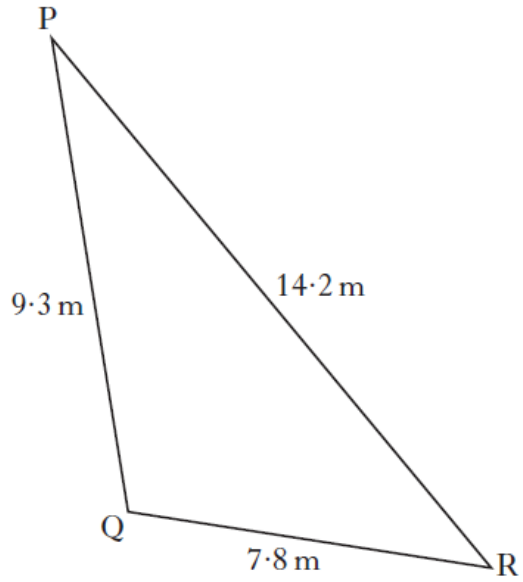
10.* An angle, $a°$, can be described by the following statements.

- a is greater than 0 and less than 360
- $\sin a°$ is negative
- $\cos a°$ is positive
- $\tan a°$ is negative

Write down a possible value for a.

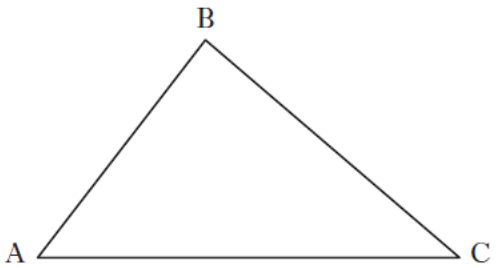
T3 – T5

1. Triangle PQR is shown below.



Calculate the size of angle QPR.

2.*



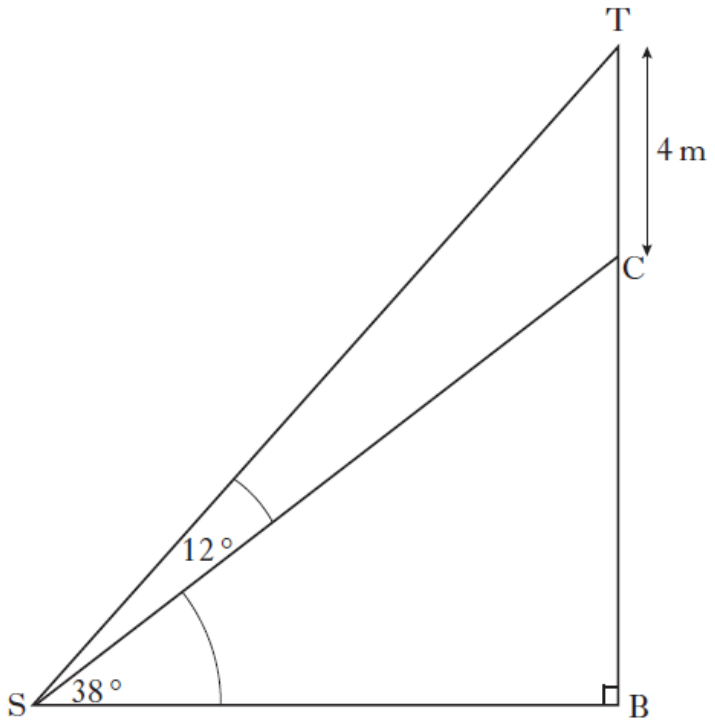
The area of triangle ABC is 20 square centimetres.

AC = 16 centimetres and $\sin C = \frac{1}{4}$.

Calculate the length of BC.

3. A tree surgeon is asked to reduce the height of a tree.

In the diagram below TB represents the original height of the tree and C is the point where the cut is to be made.



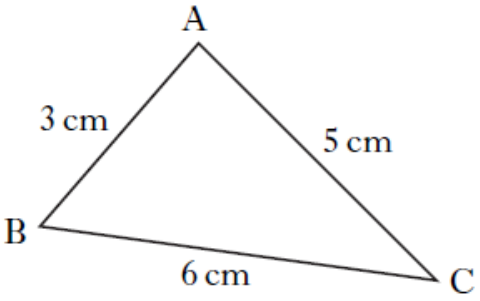
The tree surgeon will reduce the height of the tree by 4 metres.

Angle TSC = 12 ° and angle BSC = 38 °.

Calculate the height of the tree after it has been cut.

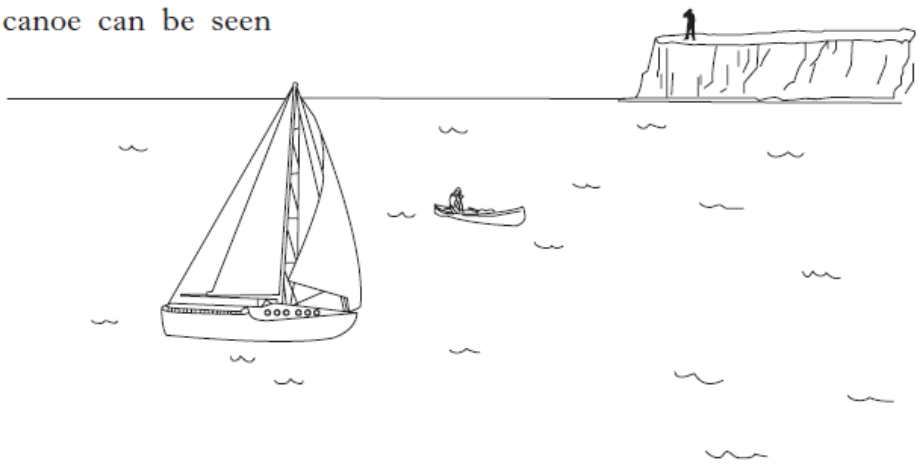
Do not use a scale drawing.

4.*

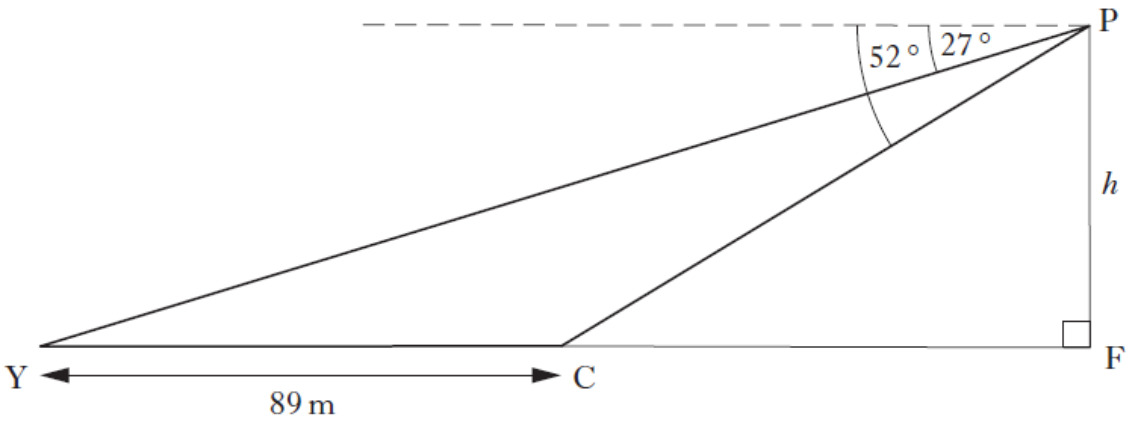


In triangle ABC, show that $\cos B = \frac{5}{9}$.

5. A yacht and a canoe can be seen from a clifftop.



In the diagram below, Y and C represent the positions of the yacht and the canoe.



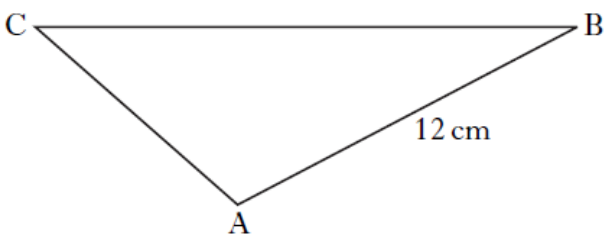
From a point P on the clifftop:

- the angle of depression of the yacht is 27 °;
- the angle of depression of the canoe is 52 °.

The distance between the yacht and the canoe is 89 metres.

Calculate the height, h, metres, of the cliff.

6.*

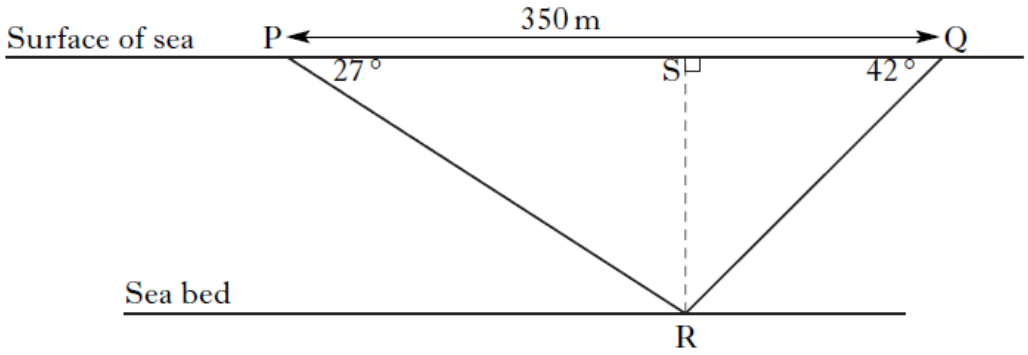


In triangle ABC, AB = 12 centimetres, $\sin C = \frac{1}{2}$ and $\sin B = \frac{1}{3}$.

Find the length of side AC.

7. Two ships have located a wreck on the sea bed.

In the diagram below, the points P and Q represent the two ships and the point R represents the wreck.



The angle of depression of R from P is 27°.
The angle of depression of R from Q is 42°.
The distance PQ is 350 metres.

Calculate QS, the distance ship Q has to travel to be directly above the wreck.

Do not use a scale drawing.

N1

1.* Simplify $\sqrt{2}\left(\sqrt{3}+\sqrt{2}\right)-\sqrt{6}$.

2.* Three of the following have the same value.

$$2\sqrt{6}, \qquad \sqrt{2}\times\sqrt{12}, \qquad 3\sqrt{8}, \qquad \sqrt{24}.$$

Which one has a different value?

You must give a reason for your answer.

3.* Express

$$\sqrt{63}+\sqrt{28}-\sqrt{7}$$

as a surd in its simplest form.

4.* Express $\dfrac{12}{\sqrt{2}}$ with a rational denominator.

Give your answer in its simplest form.

N2

1.* Simplify $\frac{x^6}{y^2} \times \frac{y^3}{x^3}$.

2.* Simplify, expressing your answer with positive indices.

$$(x^2\, y^4) \div (x^{-3}\, y^6)$$

3.* Evaluate

$$9^{\frac{3}{2}}.$$

4.* Express

$$p^3(p^2 - p^{-3})$$

in its simplest form.

5.* Simplify $\frac{ab^6}{a^3b^2}$.

N3- N5

1. A company buys machinery worth £750 000.

The value of the machinery depreciates by 20% per annum.

The machinery will be replaced at the end of the year in which its value falls below half of its original value.

After how many years should the machinery be replaced?

You must explain your answer.

2. A lead **cube**, of side 10 centimetres, is melted down.

During this process 8% of the metal is lost.

The remaining metal is then made into a **cone**, with radius 8 centimetres.

Calculate the height of this cone.

Give your answer correct to 2 significant figures.

3. The National Debt of the United Kingdom was recently calculated as

£1 157 818 887 139.

Round this amount to four significant figures.

4. Due to the threat of global warming, scientists recommended in 2010 that the emissions of greenhouse gases should be reduced by 50% by the year 2050.

The government decided to reduce the emissions of greenhouse gases by 15% **every ten years**, starting in the year 2010.

Will the scientists' recommendations have been achieved by 2050?

You must give a reason for your answer.

5. It is estimated that house prices will increase at the rate of 3·15% per annum.

A house is valued at £134 750. If its value increases at the predicted rate, calculate its value after 3 years.

Give your answer correct to **four** significant figures.

6. Michael owns shares in a bank.

His shares are currently worth £5472. This is 4% less than they were worth a year ago.

Calculate how much Michael's shares were worth a year ago?

7* Calculate $2\frac{2}{5}\times\frac{5}{8}$.

8* Calculate $1\frac{7}{8}\div 3\frac{3}{4}$.

9* Calculate $\left(\frac{3}{5}\div\frac{1}{2}-\frac{3}{4}\right)\div\frac{1}{4}$.

10* Calculate $\left(\frac{3}{4}-\frac{1}{6}\right)$ of $\frac{3}{7}$.

11* Calculate $\left(3\frac{1}{3}+\frac{5}{6}\right)\div\frac{5}{12}$.

S1 – S2

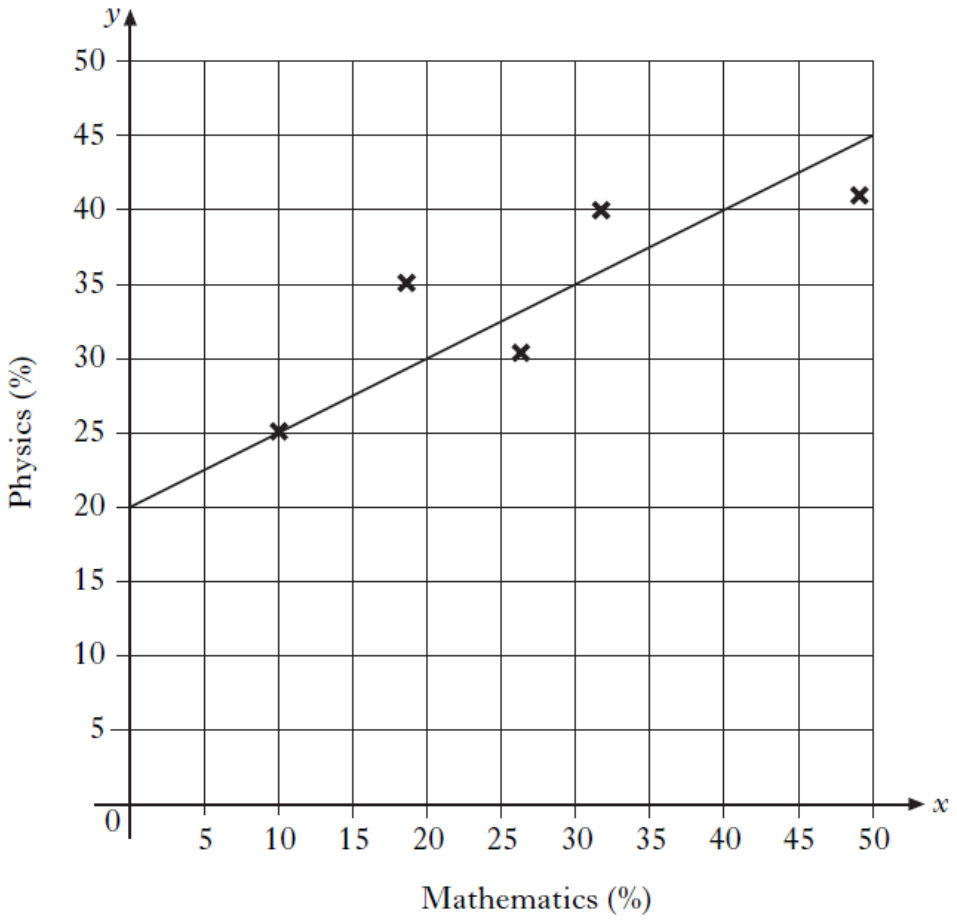
1. The results for a group of students who sat tests in mathematics and physics are shown below.

Mathematics (%)	10	18	26	32	49
Physics (%)	25	35	30	40	41

(*a*) Calculate the standard deviation for the mathematics test.

(*b*) The standard deviation for physics was 6·8.
Make an appropriate comment on the distribution of marks in the two tests.

These marks are shown on the scattergraph below.
A line of best fit has been drawn.



(*c*) Find the equation of the line of best fit.

(*d*) Another pupil scored 76% in the mathematics test but was absent from the physics test.
Use your answer to part (*c*) to predict his physics mark.

2.* (*a*) The marks of a group of students in their October test are listed below.

41 56 68 59 43 37 70 58 61 47 75 66

Calculate:

(i) the median;
(ii) the semi-interquartile range.

(*b*) The teacher arranges extra homework classes for the students before the next test in December.
In this test, the median is 67 and the semi-interquartile range is 7.

Make **two** appropriate comments comparing the marks in the October and December tests.

3. The heights, in centimetres, of seven netball players are given below.

173 176 168 166 170 180 171

For this sample, calculate:

(*a*) the mean;

(*b*) the standard deviation.

Show clearly all your working.

4. A rugby team scored the following points in a series of matches.

13 7 0 9 7 8 5

(*a*) For this sample, calculate:

(i) the mean;

(ii) the standard deviation.

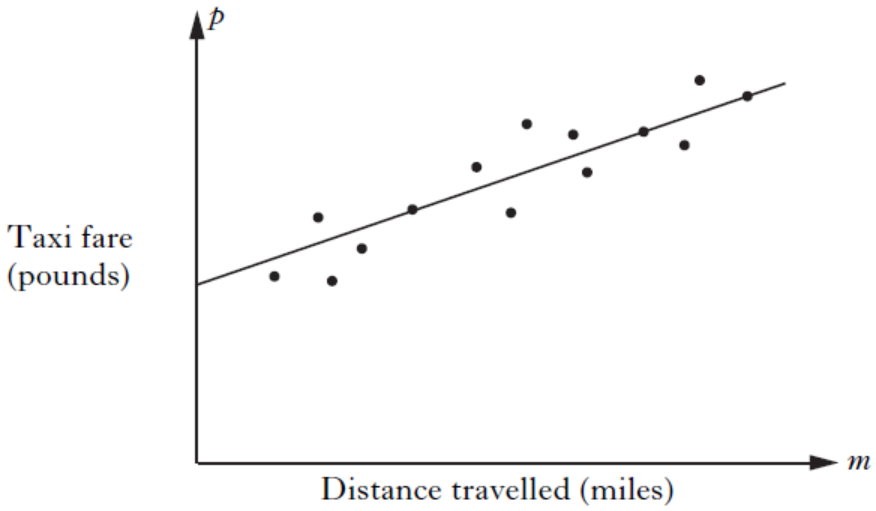
Show clearly all your working.

The following season, the team appoints a new coach.

A similar series of matches produces a mean of 27 and a standard deviation of 3·25.

(*b*) Make two valid comparisons about the performance of the team under the new coach.

5.* The scattergraph shows the taxi fare, p pounds, plotted against the distance travelled, m miles. A line of best fit has been drawn.



The equation of the line of best fit is $p = 2 + 1{\cdot}5\,m$.

Use this equation to predict the taxi fare for a journey of 6 miles.

[END OF SECTION E]